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RESEARCH ARTICLE

Cost Optimization of Solid Slabs using the Iteration Method

Abdulaziz I Almuaybid[1]*, **Abdulrazak Abdulghafour**[2], **Ehab A Mlybari**[2]

1 Master Student, Department of Civil Engineering, Umm Al-Qura, Mecca, KSA
2 Instructor, Department of Civil Engineering, Umm Al-Qura, Mecca, KSA

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* **Corresponding author**.

S44280105@st.uqu.edu.sa

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Abstract

Objectives: The research aims at minimizing construction projects' costs, including solid slab materials (steel and concrete), by analyzing the slab's parameters and optimizing the slab design using the iteration method. **Methods**: Research data were given as thirty solid slabs. Excel spreadsheets were made according to the Saudi Building Code specifications to optimize the slab design automatically and determine the minimum amount of concrete and steel. Plugging of the given data was done in an iterated method while assuring that outputs are within the allowable limits till reaching the possible optimum design. **Findings**: Study results reveal that slab 4 and slab 5 have the lowest calculated cost ($7,825.33/m^2) even though they have different amounts of concrete and steel. The two parameters, the actual depth of the slab and the steel bar' diameters, played a significant role in slab cost reduction. It is recommended that slabs from (20) to (30) be designed as flat slabs due to the chance to increase concrete instead of the steel amount. **Novelty**: The study utilized the iteration method in construction cost optimization, which the previous studies related to the topic field had not used.

Keywords: Saudi Building Code (SBC); American Concrete Institute (ACI); Design optimization; Cost minimization; Iteration method

1 Introduction

Estimating the material cost of a solid slab is related to many parameters that manipulate the cost of elements. One of these parameters is the efficiency of slab designing. Previous studies presented various methods that contribute to optimizing the design of solid slabs simultaneously with decreasing the construction cost. Multiple algorithms have been employed to optimize the cost of slabs. Choi and others [1] developed an efficient optimal design method for a steel double-beam floor system by simply providing design parameters. They suggested that the design length of steel beams be from 6.4 m to 8.2 m. Ahmadi-Aggestam and others [2] tried to minimize the environmental impact of the concrete parts while still fulfilling the standard requirements using the static calculation method. Jelusic [3] performed material cost optimization of a reinforced concrete section using the mixed integer nonlinear programming (MINLP) approach. Rady and others [4] developed a design optimization model using the evolutionary algorithm to minimize construction materials costs and labor in RC buildings. It is noticed that the construction costs could be significantly reduced by considering the concrete compressive strength and the column spacings as design variables. Kaveh and Fakoor[5] optimized floor systems supported by castellated beams using a computerized cost optimization program. It is observed that the optimum solution cannot be achieved without the integrated optimization of the main constituents of steel-concrete composite floor systems. Kumar and Akhtar[6] presented a Reinforced Concrete (RC) slab design optimization technique for finding the best design parameters that satisfy the project requirement both in terms of strength and serviceability criteria and the overall construction cost to a minimum. Elhegazy and others[7] designed a model and charts that can be used in decision-making and provided a recommendation for selecting the optimal structural system for multi-story buildings. Natarajan and others[8] optimized RC slabs using non-traditional optimization techniques such as yield line theory and Bacterial Foraging Optimization Technique. The obtained results after optimization produced an acceptable and reasonable solution. Regarding the parameters that should be focused on o reduce the concrete structure cost, such as slabs. Ranjan and others[9] presented a novel constraint to prevent local overheating for use in topology optimization (TO). The basis for the constraint is the Additive Manufacturing (AM) process, which enables the fabrication of highly complex topologically optimized designs. Zenisek and others[10] optimized six variants of reinforced concrete structures (RC) in terms of cost and environmental impact. They stated that choosing the suitable concrete strength class can save significant costs and mitigate the construction environmental impact. Kaveh and others[11] used a modified particle swarm optimization algorithm (PSO) to optimize the large-scale concrete slabs. This modification is accomplished by adding coefficients that provide the logarithm with exploration and exploitation and decrease PSO dependence on its constants. Kaveh and Bijary[12] utilized three developed algorithms, known as Particle Swarm Optimization, Democratic Particle Swarm Optimization, and Colliding Bodies Optimization, for the optimal design of a concrete ribbed slab. The study results show that DPSO and CBO methods are efficient approaches for finding the optimum solution to structural optimization problems. Kaveh and Benham[13] performed the optimum design of some floor systems, including composite slab, one-way waffle slab, and the formwork of a concrete slab, via recently developed meta-heuristic algorithms known as Charged System Search (CSS), Enhanced Charged System Search (E-CSS) and Improved Harmony Search (HIS). The results were compared to demonstrate the efficiency of the CSS algorithm. Kaveh and Shakouri[14] presented a cost optimization of a reinforced concrete one-way joist floor system consisting of a hollow slab using the harmony search algorithm. It is found that new algorithm is quite robust and efficient. Kaveh and Shakouri[15] utilized the Harmony search algorithm for the optimum design of slab formwork. The algorithm provided optimum cross-sections and spacing of the form members while minimizing the total cost. As noticed, previous studies presented various methods that contribute to optimizing the design of solid slabs simultaneously with decreasing the construction cost, but they are somehow complicated. Unlike the previous studies, this research proposed a practical method to design solid slabs with the minimum possible cost using a simple iteration method and to determine the main parameters that manipulate the slab construction cost. The method has some limitations with mega projects, such as accumulating unlimited datasets, which is found in algorithm-based methods to enhance the accuracy in the optimization process, but it is beneficial for small and medium construction projects.

2 Methodology

The study comprises a data sample of thirty slabs. We estimated the cost for each slab after doing the necessary calculations for finding steel and concrete amounts for all given slabs, which were varied in span lengths, and dead and live loads. We assumed the thicknesses and steel areas of the slabs, but lengths and loads were given. We applied analytical processes and constructed different tables that revealed a relation between two parameters, which are the bar's diameter (Ø) and slab's actual depth (d) and cost varying after plugging and assuming the required data.

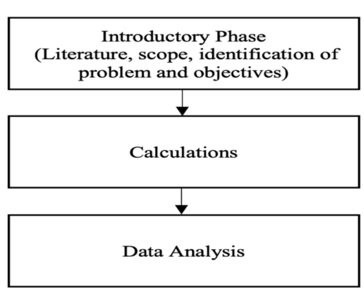

Fig 1. Methodology flowchart

Figure 1 demonstrates the process adopted to conduct the research. The introductory part focused on the literature defining and identifying the study's scope, problem, and objectives. The second phase finds the lowest amount of steel and concrete needed to estimate the cost optimization of solid slab construction.

Because the data were already given, there was little room for sample size selection for the study. The number of samples considered thirty slabs with different lengths and loads. Also, all of them were categorized as two-way solid slabs. The slabs' given parameters were short span length, long span length, unfactored dead load, and live load. The unfactored dead load and short span length were fixed in each group of spans, but the long span length and the lived load varied according to the given data.

Regarding the selective designing method, thirty excel spreadsheets were made based on the SBC and ACI code specifications. The inputs of the excel spreadsheets are short span length, long span length, unfactored dead load, unfactored live load, steel bar diameter, and the actual depth of the slab, which ranged between 100 mm to 160 mm. To control the cost of the solid slab, the actual depth and the steel bar diameter were manipulated simultaneously to avoid exceeding the preferable number of steel bars, which is ten. The ACI code specifications [16] recommended that a solid slab's thickness not be less than 100 mm and not exceed 160 mm. Additionally, it is recommended that steel bar's number not be less than Ø5@200 mm and not exceed Ø10@200 mm. According to these specifications, the optimum design of the thirty solid slabs was made. The iterative optimization method, widely used in optimizing structural design, was utilized in achieving the research objectives.

As a summary of the research followed processes in achieving slab design iterative optimization, a diagram model was constructed, as shown in Figure 2. The model represents the steps in which the slabs have been designed are:

1. Making an excel spreadsheet based on the desired code specification
2. Plugging the given known parameters

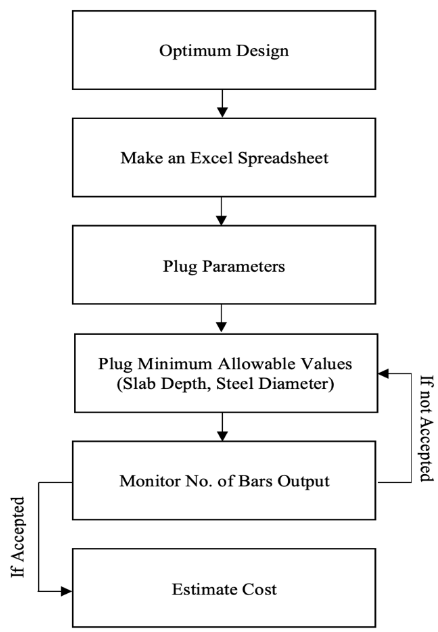

Fig 2. Diagram of Iterative Optimization Process

3. Plugging the ungiven parameters simultaneously with ensuring that the number of bars did not exceed the allowable limit and ending with estimating the cost of materials based on the current prices.

As presented by Newton in 1669, linearization is considered the basic idea of Newton's method [17]. Suppose F: R = R1 is a differentiable function, and we are solving the equation F(x) = 0. Starting from an initial point x0 we can construct the linear approximation of F(x) in the neighborhood of x0: F(x0 + h) = F(x0) + F'0(x0) h and solve the arising linear equation F(x0) + F'0(x0) h = 0. Thus, we arrive at the recurrent method

x_(k+1) = x_k - F'[(x_k) ^ (-1) F(x_k), k = 0,1,...

3 Result and Discussion

3.1 Span Length and Total Slab Cost Relationship

After tabulating the given data in an excel spreadsheet iteratively, the minimum amount of steel and concrete was provided and the cost per square meter for slab reinforcement. The specified compressive strength of concrete (F'c) was 30 MPa, while the specified steel yielding stress (Fy) was 420 MPa for all slabs. The thirty slabs were categorized into five categories according to the fixed span length.

Table 1. Material cost of reinforced concrete solid slabs with 3 m fixed span length

Slab No.	Slab Dimension (m)		Slab Area (m^2)	Unfactored Loads on Slab (KN/m^2)		Total Slab Cost	Cost/m^2
	L	W		Live Load	Dead Load		
1	3	3	9	3	3	347700	38633
2	4	3	12	4	3	502200	41850
3	5	3	15	5	3	668700	44580
4	6	3	18	6	3	528210	29345

In category 1, the fixed span length was given as 3 m. It is noticed that reinforcement cost for a 1 m^2 slab ranges from 29 SAR ($7.73) to 44.5 SAR ($11.86). These costs are in a proportional relationship to concrete and steel amounts. The lowest amount of concrete was provided with slab 1, and the lowest amount of steel was provided with slab 4. The slab with the lowest cost was slab 4, which had the lowest amount of steel and the highest amount of concrete. The lowest cost of the category's slabs was 38,633 SAR/m^2 ($10,302.13/$m^2$).

Table 2. Material cost of reinforced concrete solid slabs with 4 m fixed span length

Slab No.	Slab Dimension (m)		Slab Area (m^2)	Unfactored Loads on Slab (KN/m^2)		Total Slab Cost	Cost/m^2
	L	W		Live Load	Dead Load		
5	4	4	16	4	4	469520	29345
6	5	4	20	5	4	605380	30269
7	6	4	24	6	4	1411800	58825
8	7	4	28	7	4	1966100	70218
9	8	4	32	8	4	2234200	70100

In category 2, the fixed span length was given as 4 m. It is noticed that reinforcement cost for a 1 m^2 of slab ranges from 29 SAR ($7.73) to 70 SAR ($18.66), and these costs are related proportionally to the amount of concrete and steel. The lowest amount of concrete was provided with slab 5, and the lowest amount of steel was provided with slab 5. The slab with the lowest cost was slab 5, which had the lowest amount of steel and the lowest amount of concrete. The lowest cost of the category's slabs was 29,345 SAR/m^2 ($7,825.33/$m^2$).

Table 3. Material cost of reinforced concrete solid slabs with 5 m fixed span length

Slab No.	Slab Dimension (m)		Slab Area (m^2)	Unfactored Loads on Slab (KN/m^2)		Total Slab Cost	Cost/m^2
	L	W		Live Load	Dead Load		
10	5	5	25	5	5	1501700	60068
11	6	5	30	6	5	2106300	70210
12	7	5	35	7	5	2535547	72444
13	8	5	40	8	5	3126153	78154
14	9	5	45	9	5	3875116	86114
15	10	5	50	10	5	4763725	95275

In category 3, the fixed span length is extended to 5 m. It is noticed that the cost of reinforcement for a 1 m^2 of slab ranges from 69 SAR ($18.4) to 95 SAR ($25.33), and these costs are in a proportional relationship to the amount of concrete and steel.

The lowest amount of concrete was provided with slab 10, and the lowest amount of steel was provided with slab 10. The slab with the lowest cost was slab 10, which had the lowest amount of steel and the lowest amount of concrete. The lowest cost of the category's slabs was 60,068 SAR/m^2 (\$16,018.13/m^2).

Table 4. Material cost of reinforced concrete solid slabs with 6 m fixed span length

Slab No.	Slab Dimension (m)		Slab Area (m^2)	Unfactored Loads on Slab (KN/m^2)		Total Slab Cost	Cost/m^2
	L	W		Live Load	Dead Load		
16	6	6	36	9	6	3064800	85133
17	7	6	42	7	6	3772847	89830
18	8	6	48	8	6	5103406	106321
19	9	6	54	9	6	6462591	119678
20	10	6	60	10	6	7350790	122513
21	11	6	66	11	6	9026132	136760
22	12	6	72	12	6	10127947	140666

In category 4, the fixed span length was given as 6 m. It is noticed that reinforcement cost for a 1 m^2 of slab ranges from 85 SAR (\$22.66) to 140.5 SAR (\$37.46), and these costs are in a proportional relationship to the amount of concrete and steel. The lowest amount of concrete was provided with slab 16, and the lowest amount of steel was provided with slab 16. The slab with the lowest cost was slab 16, which had the lowest amount of steel and concrete. The lowest cost of the category's slabs was 85,133 SAR/m^2 (\$22,702.13/m^2).

Table 5. Material cost of reinforced concrete solid slabs with 7 m fixed span length

Slab No.	Slab Dimension (m)		Slab Area (m^2)	Unfactored Loads on Slab (KN/m^2)		Total Slab Cost	Cost/m^2
	L	W		Live Load	Dead Load		
23	7	7	49	7	7	6230136	127146
24	8	7	56	8	7	8490124	151609
25	9	7	63	9	7	9834636	156105
26	10	7	70	10	7	10918317	155976
27	11	7	77	11	7	12450303	161692
28	12	7	84	12	7	16330804	194414
29	13	7	91	13	7	16806440	184686
30	14	7	98	14	7	23986446	244760

In category 5, the fixed span length was given as 7 m. It is noticed that the cost of reinforcement for a 1 m^2 of slab ranges from 127 SAR (\$33.86) to 244.5 SAR (\$65.2), and these costs are in a proportional relationship to the amount of concrete and steel. The lowest amount of concrete was provided with slab 23, and the lowest amount of steel was provided with slab 23. The slab with the lowest cost was slab 23, which has the lowest amount of steel and concrete. The lowest cost of the category's slabs was 127,146 SAR/m^2 (\$33,905.6/m^2).

Table 6. Material cost of reinforced concrete solid slabs with the lowest cost in each category

Slab No.	Slab Dimension (m)		Slab Area (m^2)	Unfactored Loads on Slab (KN/m^2)		Total Slab Cost	Cost/m^2
	L	W		Live Load	Dead Load		
4	6	3	18	6	3	528210	29345
5	4	4	16	4	4	469520	29345
10	5	5	25	5	5	1501700	60068
16	6	6	36	9	6	3064800	85133
23	7	7	49	7	7	6230136	127146

Table 6 shows that the solid slabs with the lowest cost were slab 4 and slab 5 even though their concrete and steel amounts differ. From the previous tables, we can conclude that slab cost is proportionally related to the concrete and steel amount, but it is not necessary to have a proportional relation with slab reinforcement.

In a trial to summarize the previous processes, there were many trials to reach the minimum cost of steel and concrete. In case of hesitation between decreasing the steel amount and increasing the concrete amount or increasing the steel amount and decreasing the concrete amount, the first decision was chosen immediately.

It is noticed that in slabs from (1) to (19), there was an opportunity to change the amount of concrete by decreasing or increasing the slab thickness without exceeding the recommended thickness for the solid slab, 16 mm. On the contrary, the opportunity to vary the concrete amount in slabs from (20) to (30) was nonexistent. For these slabs from (20) to (30), the amount of steel was increased gradually, and because of that, it would be better if they were designed as flat slabs to have the opportunity of increasing the slab depth, which will provide a chance to increase concrete amount instead of increasing steel amount as well as reaching the minimum cost of slab construction.

3.2 Span Length and Slab Average Cost Per Square Meter Relationship

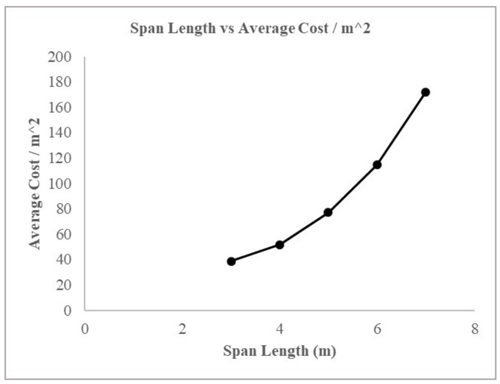

Fig 3. Span Length vs Average Cost /m^2

Figure 3 shows a proportional relation between the span length and the average cost per square meter of slab reinforcement. It was noticed that as the span length of the slab increases, the average cost per square meter increases. In addition, a strange behavior occurred in the graph curve, which is an abnormal increasing ratio (> 48%) for the reinforcement cost per square meter starting from the span length (6 m).

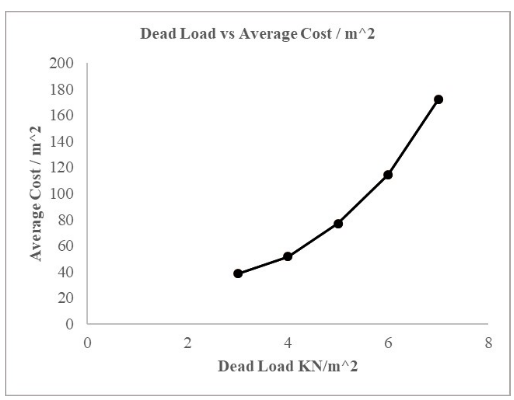

Fig 4. Dead Load vs Average Cost /m^2

Figure 4 also shows a proportional relation between the dead load and the reinforcement cost per square meter. It was noticed that as the dead load increases, the reinforcement average cost per square meter increases. In addition, a strange behavior occurred in the graph curve, which is an abnormal increasing ratio (> 48%) for the reinforcement cost per square meter starting from the span length (6 m).

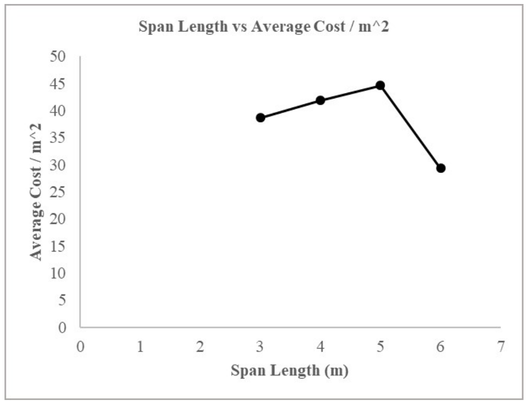

Fig 5. Slab width (3m) Span Length vs Average Cost /m^2

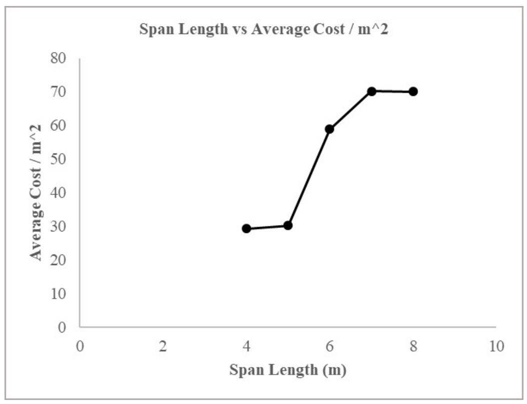

Fig 6. Slab width (4m) Span Length vs Average Cost m^2.

Figure 5 reveals an identical relation between the span length and the cost per square meter of slab reinforcement. It was noticed that the reinforcement cost increased gradually starting from slab (1), but in slab (4), the cost decreased abnormally due to an increase in the concrete amount and a decrease in the steel amount.

Figure 6 also reveals an identical relation between the span length and the cost per square meter of slab reinforcement. It was noticed that the reinforcement cost increased slightly but starting from slab (6) the cost increased abnormally due to an increase in the steel amount.

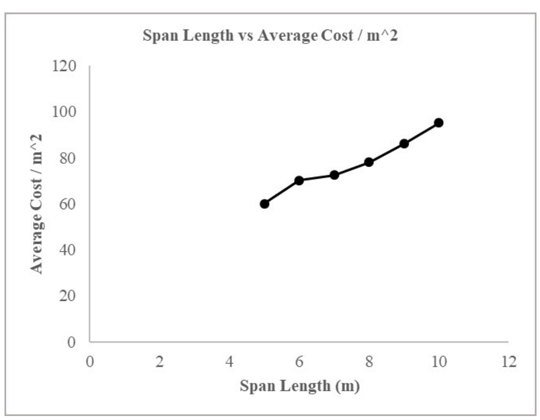

Fig 7. Slab width (5m) Span Length vs Average Cost / m^2.

Figure 7 reveals an identical relation between the span length and the cost per square meter of slab reinforcement. It was noticed that the reinforcement cost increased gradually, but in slab 12 the cost decreased slightly due to an increase in the concrete amount and a decrease in the steel amount.

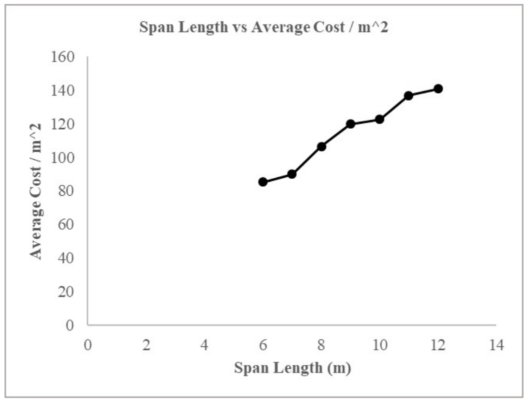

Fig 8. Slab width (6m) Span Length vs Average Cost / m^2.

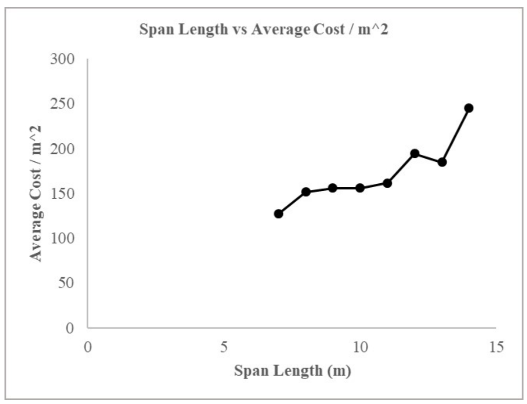

Fig 9. Slab width (7m) Span Length vs Average Cost / m^2.

Figure 8 reveals an identical relation between the span length and the cost per square meter of slab reinforcement. It was noticed that the reinforcement cost almost increased gradually.

Figure 9 reveals an identical relation between the span length and the cost per square meter of slab reinforcement. It was noticed that the reinforcement cost increased gradually, but in slabs (26) and (29) the reinforcement cost exceptionally decreased due to an increase in the concrete amount and a decrease in the steel amount.

As shown from the previous figures, utilizing the iterative method in the optimization process provides basic knowledge about the relationship between the construction parameters and the total construction cost. This knowledge helps determine the suitable type of construction part that assures getting the minimum cost of construction project using cheap and non-complicated methods as mentioned in the literature previously.

3.2.1 Span Length and Total Slab Cost

Slab thickness is a vital factor in building design. It is directly related to the cost of the structural system. Increasing slab thickness leads to an increase in the column axial loads and sizes. So, it is crucial to limit the slab thickness to the limits required by design. During the design process, we tried to decrease the slab thickness to the minimum while considering the design requirements limits, and most of the slab thicknesses were 160 mm.

3.3 Maximum Bending Moment and Slab Average Cost Per Square Meter Relationship

It is noticed that the relationship between the maximum bending moment values and the cost per square meter for the given slabs forms a linear curve. It indicates a positive proportional relationship between the cost per square meter, the value of the total factored load carried by the slab and the length of the slab span. The relationship is represented by Equation (1):

$$y = 0.97x + 21.9 \tag{1}$$

where,

y: the cost per square meter (SAR/m^2)

x: the value of the maximum bending moment.

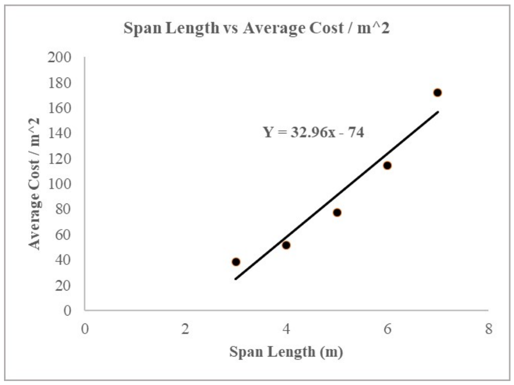

Fig 10. Maximum Bending Moment vs Cost / m^2

4 Conclusion

This article investigates the methods used to optimize the design of solid slabs and provides a practical method for optimizing the design of solid slabs. This method depends on excel spreadsheets to reach the optimum design and minimum cost of solid slab construction. The main parameters that were manipulated in the slab cost are the bar's diameter (Ø) and the slab's actual depth (d). The thirty slabs were divided into five categories based on the fixed span length for the given slabs. After designing the thirty slabs and estimating the cost of each, various tables were constructed. In summary, for the design optimization process outputs, the least slab cost in category 1 was 38,633 SAR/m^2 ($10,302.13/m^2), which has the least steel amount and the highest concrete amount. While in category 2, the lowest slab cost was 29,345 SAR/m^2 ($7,825.33/m^2), which has the minimum amount of steel and concrete. Additionally, the lowest slab cost was 60,068 SAR/m^2 ($16,018.13/m^2) in category 3, which has the lowest steel and concrete amounts. Moreover, the lowest slab cost was 85,133 SAR/m^2 ($22,702.13/m^2) in category 4, which has the lowest steel and concrete amounts, and the lowest slab cost was 127,146 SAR/m^2 ($33,905.6/m^2) in category 5, which also has the lowest of steel and concrete amounts. Slabs from (1) to (19) were offered an opportunity to change the amount of concrete without exceeding the recommended thickness for the solid slab, which is 160 mm, unlike slabs from (20) to (30) that were not provided the same opportunity. As a result of the analysis of the outputs, it is recommended for slabs from (20) to (30) to be designed as flat slabs due to the chance to increase concrete amount instead of increasing the steel amount. A positive relationship between the cost per square meter and the total factored load and span lengths is represented by the following equation: y = 0.97x + 21.9, where y is the cost per square meter and x is the value of the maximum bending moment. The article's findings are introductory for engineers to be more aware of not neglecting the optimization process while designing future projects. Also, to utilize the various algorithms in the optimization process and compare their results with the results of the iteration method to identify the most suitable optimization method for the particular case.

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