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RESEARCH ARTICLE

Free Vibrations in a Rotating Generalized Elastic Hollow Solid Sphere

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Abstract

Objective: To investigate the free vibrations in a rotating elastic hollow solid sphere. **Method:** The method of plane harmonic solution is employed to solve the basic governing equations of rotating elastic solid. **Findings:** Three types of frequency equations named as coupled frequency (CF) (Radial and tangential) equations, radial frequency (RF) equation and tangential frequency (TF) equations are derived. **Novelty:** Under the MATLAB program, the numerical computations have been performed for a particular material. Frequency versus angle is shown graphically for rotating and non-rotating material. TF and CF curves are inverse proportional to the angular rotation. Coupled frequencies are slower than the tangential frequencies.

Keywords: Free Vibrations; Rotation; Hollow sphere; Radial frequencies; Tangential frequencies and Coupled Frequencies

1 Introduction

The study of free vibrations in a generalized elastic solid has been a subject of extensive investigation in the literature. It is of great importance of variety applications in many engineering fields like Aerospace, Civil, Mechanical, Navel, Chemical and Nuclear Engineering. The generalized theory of elasticity has drawn widespread attention because it removes the physically unacceptable situation of the classical theory of elasticity. Some of spherical or a part of spherical shape structures are saturated soil, osseous tissues, sedimentary rocks and human body. The generalized theory of elasticity such as thermo elasticity was developed by Lord-Shulman[(1)],Green –Lindsay[(2)]. The effect of non-locality on free vibrations in a thermo-elastic hollow cylinder with diffusion was studied by Dinesh Kumar Verma et.al.[(3)]. Forced axisymmetric vibrations in an inhomogeneous piezoceramic hollow sphere are investigated by Grigorenko and Loza[(4)]. Hamdy[(5)] studied the effects of mechanical damage, radial distance, diffusion on Lord-Shulman's thermo elastic sphere. Eman[(6)] presented vibration analysis of a nanobeam due to a ramp type heating under Moore-Gibson-Thompson theory of thermo-elasticity. Free vibrations in the visco-thermo elastic hollow sphere are presented by Dinesh Kumar sharma and Himani Mittal[(7)] .

Some of authors like Somaiah[8] and Somaiah and Chandulal[9] arestudied the rotation effect on radial vibrations, and inhomogeneous and attenuation waves respectively.

This present paper is arranged in the following manner. Governing equations are presented and solved in section 2, results and discussion are given with the help of MATLAB software in section 3 and overall conclusion in section 4.

2 Methodology

With the usual notations of Eringen[10] the constitutive equations and field equations of rotating elastic solid in the absence of body forces are

$$(\lambda+2\mu+K)\nabla\nabla\cdot\vec{u}-(\mu+K)\nabla\times(\nabla\times\vec{u})=\rho\left[\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2(\vec{\Omega}\times\dot{\vec{u}})\right] \tag{1}$$

$$t_{ij}=\lambda e_{kk}\delta_{ij}+(2\mu+K)e_{ij} \tag{2}$$

where $\lambda,\ \mu$ Lame's parameters, K is a material constant, δ_{ij} is the Kroneckers delta, ρ is the density of the material. The super pose dot is the partial differentiation with respect to time t, $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})$ is Centripetal acceleration and $2(\vec{\Omega}\times\dot{\vec{u}})$ is the Coriolis acceleration and

$$e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \tag{3}$$

We consider a homogeneous rotating elastic hollow solid sphere of inner radius a and outer radius b. We use the spherical coordinates (r,θ,Φ). The centre of the sphere taken as origin of the coordinate system and sphere is rotating about z-axis with the angular velocity $\vec{\Omega}$. So the macro displacement vector $\overrightarrow{\mu}$ taken as $\overrightarrow{u}=(u_r,u_\theta,0)$ where $u_{r=}u_r(r,\theta,t),u_\theta=u_\theta(r,\theta,t)$ are respectively the displacements components in radial and tangential directions. Angular velocity $\vec{\Omega}$ taken as $\vec{\Omega}=(0,0,\Omega)$.

In this case, $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2(\vec{\Omega}\times\dot{\vec{u}})=\left(-\Omega^2u_r-2\Omega\dot{u}_\theta,-\Omega^2u_\theta+2\Omega\dot{u}_r,0\right)$.

With the help of equation (3) and fundamental vector calculus, the equations (1) and (2) reduces in the directions of r,θ as

$$a_1\left(e_1u_r+e_2u_\theta\right)+a_2\left(e_3u_r-e_4u_\theta\right)=\rho\left[\ddot{u}_r-\Omega^2u_r-2\Omega\dot{u}_\theta\right] \tag{4}$$

$$a_1\left(e_5u_r+e_6u_\theta\right)+a_2\left(e_7u_\theta-e_8u_r\right)=\rho\left[\ddot{u}_\theta-\Omega^2u_\theta+2\Omega\dot{u}_r\right] \tag{5}$$

$$t_{rr}=(\lambda+2\mu+K)u_{r,r}+\lambda u_{\theta,\theta},\quad t_{r\theta}=\mu u_{r,\theta}+(\mu+K)u_{\theta,r} \tag{6}$$

where ; $a_1=\lambda+2\mu+K; a_2=\mu+K$

$$\begin{aligned}
&e_1=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{2}{r^2};\quad e_2=\frac{1}{r}\left(\frac{\partial^2}{\partial r\partial\theta}+\cot\theta\frac{\partial}{\partial r}\right)-\frac{1}{r^2}\left(\frac{\partial}{\partial\theta}+\cot\theta\right)\\
&e_3=\frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2}+\cot\theta\frac{\partial}{\partial\theta}\right);\quad e_4=\frac{2}{r^2}\left(\frac{\partial}{\partial\theta}+\cot\theta\right)+e_2;\quad e_5=\frac{1}{r}\left(\frac{\partial^2}{\partial\theta\partial r}+\frac{2}{r}\frac{\partial}{\partial\theta}\right);\\
&e_6=\frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2}+\cot\theta\frac{\partial}{\partial r}-\operatorname{cosec}^2\theta\right),\quad e_7=\frac{\partial^2}{\partial r^2}-\frac{2}{r}\frac{\partial}{\partial r};\quad e_8=\frac{1}{r}\frac{\partial^2}{\partial\theta\partial r};
\end{aligned} \tag{7}$$

To study the propagation of harmonic waves, we seek the solution of the form

$$u_r(r,\theta,t)=Ae^{m_1r}e^{i(q\theta-\omega t)},\quad u_\theta(r,\theta,t)=Be^{m_2r}e^{i(q\theta-\omega t)} \tag{8}$$

where A and B are arbitrary constants, q is the wave number, ϖ is the angular frequency, $q=\frac{2\pi}{l}$, where l is the wave length and

$$m_1=\frac{1}{2}\left[-X_1\pm\left(X_1^2-4X_2\right)^{\frac{1}{2}}\right],\quad m_2=\frac{1}{2}\left[-Y_1\pm\left(Y_1^2-4Y_2\right)^{\frac{1}{2}}\right] \tag{9}$$

$$\begin{aligned} & X_1=2r^{-1}a^{-1}-iqr^{-1}\left(1-a_2a_1^{-1}\right), X_2=a_1^{-1}\left[c_1+\rho\left(\omega^2+\Omega^2+2i\,q\Omega\right)\right]-c_2, \\ & Y_1=a_1a_2^{-1}\left[d_1+iqr^{-1}\right]+2a_2^{-1}r^{-1}+d_2+iqr^{-1} \\ & Y_2=\rho a_2^{-1}\left(\omega^2+\Omega^2-2i\,q\Omega\right)-q^2r^{-2}a_2^{-1}+a_1a_2^{-1}b_1+b_2, \\ & b_1=r^{-2}\cot\theta+iq-r^{-2}\left(\operatorname{cosec}c^2\theta+1\right), b_2=r^{-2}(\cot\theta+iq), \\ & c_1=a_2r^{-2}\left(iq\cot\theta-q^2\right), c_2=2\left(r^{-2}+iqr^{-1}\right) \\ & d_1=(i-r)r^{-2}\cot\theta, d_2=r^{-1}\cot\theta \end{aligned} \tag{10}$$

Because of equation (9), the solutions (8) reduces of the form

$$u_r(r,\theta,t)=\left[A_1e^{p_1r}+A_2e^{p_2r}\right]e^{i(q\theta-\omega t)} \tag{11}$$

$$u_\theta(r,\theta,t)=\left[B_1e^{q_1r}+B_2e^{q_2r}\right]e^{i(q\theta-\omega t)} \tag{12}$$

where A_1, A_2, B_1, B_2 are arbitrary constants and

$$\begin{aligned} & p_1=\frac{1}{2}\left[-X_1+\left(X_1^2-4X_2\right)^{\frac{1}{2}}\right]; p_2=\frac{1}{2}\left[-X_1-\left(X_1^2-4X_2\right)^{\frac{1}{2}}\right] \\ & q_1=\frac{1}{2}\left[-Y_1+\left(Y_1^2-4Y_2\right)^{\frac{1}{2}}\right]; q_1=\frac{1}{2}\left[-Y_1-\left(Y_1^2-4Y_2\right)^{\frac{1}{2}}\right] \end{aligned} \tag{13}$$

2.1 Boundary Conditions and Frequency Equations

The solutions of the hollow sphere with different boundary conditions are performed, the mixed boundary conditions which consist of two kinds of boundary conditions, the inner surface fixed and the outer surface free i.e.,

$$\begin{aligned} & u_r=u_\theta=0 \quad \text{at } r=a \\ & t_{rr}=0, t_{r\theta}=0 \quad \text{at } r=b \end{aligned} \tag{14}$$

Inserting equations (11) and (12) in equation (14), we obtain the following system of homogeneous equations in A_1, A_2, B_1, B_2

$$\begin{aligned} & A_1e^{p_1a}+A_2e^{p_2a}=0, B_1e^{q_1a}+B_2e^{q_2a}=0 \\ & A_1a_1p_1e^{p_1b}+A_2a_1p_2e^{p_2b}+B_1\lambda i\,qe^{q_1b}+B_2\lambda i\,qe^{q_2b}=0 \\ & A_1i\,\mu qe^{p_1b}+A_2i\,\mu qe^{p_2b}+B_1a_2q_1e^{q_1b}+B_2a_2q_2e^{q_2b}=0 \end{aligned} \tag{15}$$

The system (15) has non-trivial solutions if and only if

$$\left|a_{ij}\right|=0; i,j=1,2,3,4 \tag{16}$$

where

$$\begin{aligned} & a_{11}=e^{p_1a}; a_{12}=e^{p_2a}; a_{13}=a_{14}=a_{21}=a_{22}=0 \\ & a_{23}=e^{q_1a}; a_{24}=e^{q_2a}; a_{31}=a_1p_1e^{p_1b}; a_{32}=a_1p_2e^{p_2b}; \\ & a_{33}=\lambda i\,qe^{q_1b}; a_{34}=\lambda i\,qe^{q_2b}; a_{41}=\mu i\,qe^{p_1b}; \\ & a_{42}=\mu i\,qe^{p_2b}; a_{43}=a_2q_1e^{q_1b}; a_{44}=a_2q_2e^{q_2b} \end{aligned} \tag{17}$$

The equation (16) is the coupled (radial and tangential) dispersion relation of free vibrations in a rotating hollow sphere.

Special Cases:

(i) When m_1 vanishes i.e., $p_1=p_2=0$, at $r=a, r=b$ we obtain the dispersion along the tangential direction is given by

$$\frac{\omega^2+\left(\Omega^2-2i\,q\Omega\right)}{\rho a_2^{-1}}=a_1a_2^{-1}\left(d_1-b_1\right)+\left(d_2-b_2\right)+a_2^{-1}r^{-1}\left(q^2r^{-1}+a_1i\,q+2\right) \tag{18}$$

(ii) When m_2 vanishes, i.e.,, $q_1=q_2=0$, at $r=a, r=b$ we obtain the dispersion along the radial direction is

$$\frac{\rho\omega^2}{a_1}=X_1+c_2-a_1^{-1}\left[c_1a_2r^{-1}+\rho\left(\omega^2+2i\,\Omega q\right)\right] \tag{19}$$

3 Results and Discussion

Under the theoretical illustrations presented in the previous sections, we now present some numerical results. The physical data of the magnesium crystal - like material which modelled as generalized elastic solid is given by Sharma et.al[11]; $\lambda = 2.17 \times 10^{10}\ \mathrm{N/m^2}; \mu = 1.639 \times 10^{10}\ \mathrm{N/m^2}; \rho = 1.74 \times 10^3\ \mathrm{kg/m^3}; K = 1.7 \times 10^2\ \mathrm{W/m_{deg}}$

The values of angle θ in degree taken as $10^0 \le \theta \le 240^\circ$. For computational purpose the radius has been taken r = 10 cm, natural wave number q of the solid taken as $q = 2\pi/l$ for wave length l = 10cm. The angular rotation Ω selected as; rotation $\mathrm{I} = 0.5 \times 10^3$rps, rotation $\mathrm{II} = 1 \times 10^3$ rps and rotation $III = 5 \times 10^3$ rps. Using MATLAB software, numerical computations have been performed and variation of frequency for angle are plotted for non rotation and angular rotation I, II and III.

The coupled frequency (CF) curves for non rotation and rotations I, II and III in the given range of angle θ are shown in figure (1). From this figure we observed that coupled frequencies in non-rotating material are faster than in rotating material.

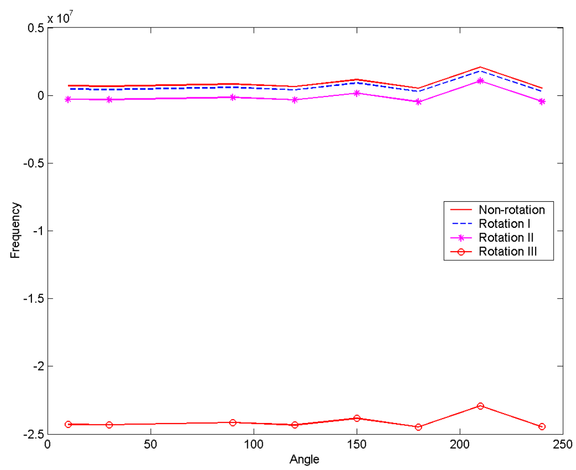

Fig 1. Coupled Frequency versus angle

Tangential frequencies are shown in figure (2). Also tangential waves in non-rotating material are faster than in rotating material. Coupled and tangential waves are rapidly jumped at 210^0

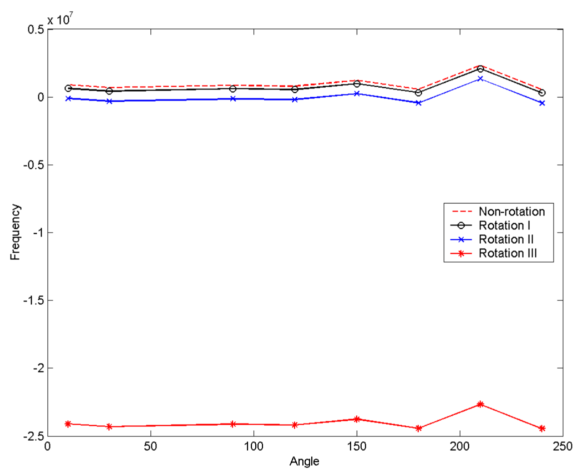

Fig 2. Tangential Frequency versus angle

The comparative CF and TF curves are shown for non-rotation and the rotations-I, II, III in figures (3) to (6). All the frequency curves are compared in figure (7).

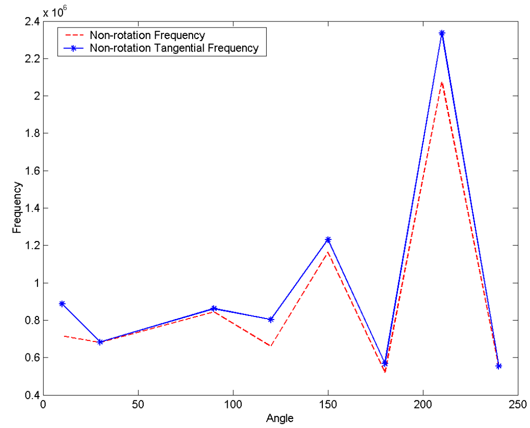

Fig 3. Comparative CF and TF curves

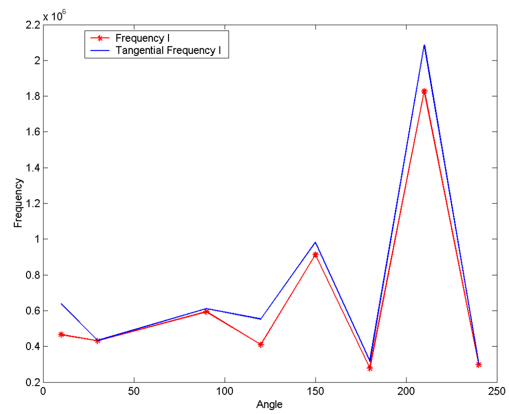

Fig 4. Comparative CF and TF curves

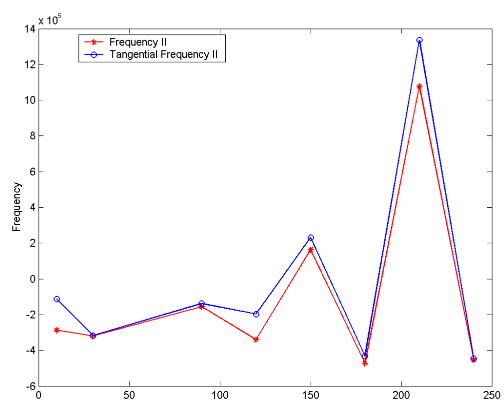

Fig 5. Comparative CF and TF curves

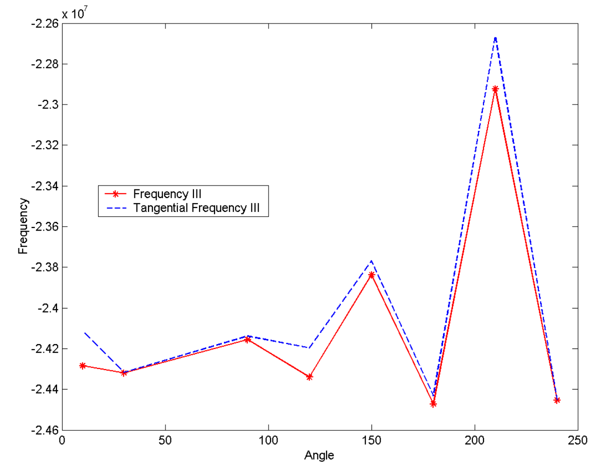

Fig 6. Comparative CF and TF curves

From these figures we observed that tangential waves are faster than coupled waves in rotating and non-rotating solid materials.

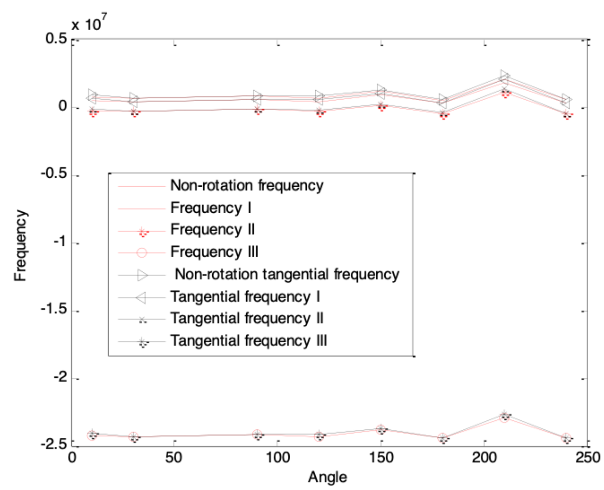

Fig 7. All the frequency curves

From the above graphical study we observed that CF and TF are inverse proportional to the angular rotation of the solid. TF is faster than to the CF in the non-rotating and rotating solid for free vibrations. All these frequency curves are rapidly jumped at $\theta = 210°$.

4 Conclusion

For deriving free vibrations in a hollow solid sphere, the basic equations are converted into spherical coordinates and solved by the method of plane harmonic solution. Three types of vibrations named as radial, tangential and coupled (radial and tangential) are derived for hollow solid sphere. The effect of rotations on free vibrations also discussed. Also we observed that:

- Tangential and coupled vibrations in a non-rotating solid are faster than in rotating solid
- Tangential vibrations are faster than coupled vibrations
- Tangential and coupled vibrations are inverse proportional to the angular rotations of the solid
- The frequencies are rapidly jumped at 210^0

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