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A Newly Proposed and Comparative Study on Multi - Criteria Decision Making in Interval Valued Bipolar Triangular Neutrosophic Set by PROMETHEE II

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Abstract

Objectives: This paper propose the operational laws and operators in interval valued bipolar triangular neutrosophic sets and also to resolve multi criteria decision making problem using PROMETHEE II. **Methods:** Based on the combination of bipolar neutrosophic sets and interval valued triangular neutrosophic sets, an interval valued bipolar triangular neutrosophic set is proposed followed by its operational laws, score function, accuracy and certainty function. **Findings:** The arithmetic operators, operational laws and also the score function, accuracy and certainty functions have been proposed for interval valued bipolar triangular neutrosophic sets. The PROMETHEE II technique discovered and employed an aggregated weighted arithmetic operator to address the multi-criterion decision-making problem. **Novelty:** By the concept of interval valued bipolar triangular neutrosophic set, the results can be viewed with more accuracy for the linguistic variables rather than the crisp values. An illustrative example has been explained for the interval valued bipolar triangular neutrosophic numbers by which the effectiveness is tested with the comparative approach by using PROMETHEE II method. **Keywords :** Interval valued neutrosophic numbers, Bipolar neutrosophic numbers, Interval valued bipolar triangular neutrosophic numbers, Score function, PROMETHEE II method.

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1 Introduction

Smarandache [1] in 1998 proposed the neutrosophic set(NS), is a very powerful mathematical tool to deal with vague, incomplete and imprecise data which we often face in today's scenario. It is a generalization of the fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets, interval valued intuitionistic fuzzy sets. Neutrosophic sets are characterized by truth membership function(T), an indeterminacy membership function(I) and falsity membership function(F) which takes the values from $]0^-, 1^+[$.

Lee [2] proposed Bipolar fuzzy sets which is the generalization of fuzzy sets of Zadeh's [3]. Bosc and Pivert [4] quoted that "Bipolarity refers to the prosperity of the human kind to make reasons and make decisions on the basis of both positive and negative effects".

2 Preliminaries

Definition 2.1. [5] "Let X be the universe of discourse. Then a single valued neutrosophic set is defined as: $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle : x \in X\}$ which is characterized by a truth-membership function $T_A(x) : X \to [0,1]$, an indeterminacy-membership function $I_A(x) : X \to [0,1]$, and a falsity-membership function $F_A(x) : X \to [0,1]$ and the sum $T_A(x), I_A(x)$ and $F_A(x)$ should satisfy the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$".

Definition 2.2. [6]" **An** interval valued NS is defined by

$$\widetilde{N} = \left\{ < x : [T_{\widetilde{N}^l(x), T_{\widetilde{N}^u(x)}}], \left(I_{\widetilde{N}^l(x), I_{\widetilde{N}^u(x)}} \right], \left([F_{\widetilde{N}^l(x), F_{\widetilde{N}^u(x)}} \right] > x \in X \right\}, \text{ whereas}$$

$$T_{\widetilde{N}(x) = \left[T_{\widetilde{N}^l(x), T_{\widetilde{N}^u(x)}} \right] \subseteq [0,1]},$$

$$I_{\widetilde{N}}(x) = \left[I_{\widetilde{N}^l(x), I_{\widetilde{N}^u(x)}} \right] \subseteq (0,1],$$

$$F_{\widetilde{N}}(x) = \left[F_{\widetilde{N}^l(x), F_{\widetilde{N}^u(x)}} \right] \subseteq (0,1] \text{ and } 0 \le sup\ T_{\widetilde{N}}(x) + sup\ I_{\widetilde{N}}(x) + sup\ F_{\widetilde{N}}(x) \le 3".$$

Definition 2.3. [7] "A bipolar neutrosophic set $\bar{A}$ in X is defined as an object of the type $\bar{A} = \{\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)\rangle : X\}$,where $T^+, I^+, F^+ : X \to [0,1]$ and $T^-, I^-, F^- : X \to [-1,0]$. The positive membership degree $T^+(x), I^+(x), F^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set $\bar{A}$, while the negative membership degree $T^-(x), I^-(x), F^-(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set $\bar{\bar{A}}$in X.

Definition 2.4. [8] "An interval valued bipolar neutrosophic set $\bar{\bar{A}}$ in X is defined as an object of the form

$$\bar{\bar{A}} = \left\{ \left[T_l^+(x), T_u^+(x)\right], \left[I_l^+(x), I_u^+(x)\right], \left[F_l^+(x), F_u^+(x)\right], \left[T_l^-(x), T_u^-(x)\right], \left[I_l^-(x), I_u^-(x)\right], \left[F_l^-(x), F_u^-(x)\right] \right\}$$

where $T_l^+, T_u^+, I_l^+, I_u^+, F_l^+, F_u^+ : X \to [0,1]$ and $T_l^-, T_u^-, I_l^-, I_u^-, F_l^-, F_u^- : X \to [-1,0]$".

Definition 2.5. [9] "Let us consider

$$\bar{\bar{A}} = \left\{ \left[T_l^+(x), T_u^+(x)\right], \left[I_l^+(x), I_u^+(x)\right], \left[F_l^+(x), F_u^+(x)\right], \left[T_l^-(x), T_u^-(x)\right], \left[I_l^-(x), I_u^-(x)\right], \left[F_l^-(x), F_u^-(x)\right] \right\}$$

and

$$\bar{\bar{A}} \left\{ \left[T_{l2}^+(x), T_{u2}^+(x)\right], \left[I_{l2}^+(x), I_{u2}^+(x)\right], \left[F_{l2}^+(x), F_{u2}^+(x)\right], \left[T_{l2}^-(x), T_{u2}^-(x)\right], \left[I_{l2}^-(x), I_{u2}^-(x)\right], \left[F_{l2}^-(x), F_{u2}^-(x)\right] \right\}$$

be two interval valued bipolar neutrosophic sets, then

1. $\bar{\bar{A}}_1 \subseteq \bar{\bar{A}}_2$ iff
$T_{l1}^+(x) \le T_{l2}^+(x), T_{u1}^+(x) \le T_{u2}^+(x), I_{l1}^+(x) \ge I_{l2}^+(x),\ I_{u1}^+(x) \ge I_{u2}^+(x), F_{l1}^+(x) \ge F_{l2}^+(x),$
$F_{u1}^+(x) \ge F_{u2}^+(x)$, and
$T_{l1}^-(x) \le T_{l2}^-(x), T_{u1}^-(x) \le T_{u2}^-(x), I_{l1}^-(x) \ge I_{l2}^-(x),\ I_{u1}^-(x) \ge I_{u2}^-(x), F_{l1}^+(x) \ge F_{l2}^-(x),$
$F_{u1}^-(x) \ge F_{u2}^-(x)$ for all $x \in X$.

2. $\bar{\bar{A}}_1 = \bar{\bar{A}}$ iff
$T_{l1}^+(x) = T_{l2}^+(x), T_{u1}^+(x) = T_{u2}^+(x), I_{l1}^+(x) = I_{l2}^+(x),\ I_{u1}^+(x) = I_{u2}^+(x), F_{l1}^+(x) = F_{l2}^+(x),$

$F_{u1}^{+}(x)=F_{u2}^{+}(x)$, and$T_{l1}^{-}(x)=T_{l2}^{-}(x), T_{u1}^{-}(x)=T_{u2}^{-}(x), I_{l1}^{-}(x)=I_{l2}^{-}(x)$, $I_{u1}^{-}(x)=I_{u2}^{-}(x), F_{l1}^{+}(x)=F_{l2}^{-}(x)$, $F_{u1}^{-}(x)=F_{u2}^{-}(x)$ for all $x\in X$.

3. The union of $\overline{\overline{A}}_1$ and $\overline{\overline{A}}_2$ is defined as,

$$\left(\overline{\overline{A}}_1\cup\overline{\overline{A}}_2\right)(x)=$$

$$\left(\begin{array}{l}\left[\max\left[T_{l1}^{+}(x),T_{l2}^{+}(x)\right],\max\left[T_{u1}^{+}(x),T_{u2}^{+}(x)\right]\right],\left[\min\left[I_{l1}^{+}(x),I_{l2}^{+}(x)\right],\min\left[I_{u1}^{+}(x),I_{u2}^{+}(x)\right]\right],\\ \left[\min\left[F_{l1}^{+}(x),F_{l2}^{+}(x)\right],\min\left[F_{u1}^{+}(x),F_{u2}^{+}(x)\right]\right],\\ \left[\min\left[T_{l1}^{-}(x),T_{l2}^{-}(x)\right],\min\left[T_{u1}^{-}(x),T_{u2}^{-}(x)\right]\right],\left[\max\left[I_{l1}^{-}(x),I_{l2}^{-}(x)\right],\max\left[I_{u1}^{-}(x),I_{u2}^{-}(x)\right]\right],\\ \left[\max\left[F_{l1}^{-}(x),F_{l2}^{-}(x)\right],\max\left[F_{u1}^{-}(x),F_{u2}^{-}(x)\right]\right]\end{array}\right) \text{ for all } x\in X.$$

4. The intersection of $\overline{\overline{A}}_1$and $\overline{\overline{A}}_2$ is defined as,

$$\left(\overline{\overline{A_1}}\cap\overline{\overline{A}}_2\right)(x)=$$

$$\left(\begin{array}{l}\left[\min\left[T_{l1}^{+}(x),T_{l2}^{+}(x)\right],\min\left[T_{u1}^{+}(x),T_{u2}^{+}(x)\right]\right],\left[\max\left[I_{l1}^{+}(x),I_{l2}^{+}(x)\right],\max\left[I_{u1}^{+}(x),I_{u2}^{+}(x)\right]\right],\\ \left[\max\left[F_{l1}^{+}(x),F_{l2}^{+}(x)\right],\max\left[F_{u1}^{+}(x),F_{u2}^{+}(x)\right]\right],\\ \left[\max\left[T_{l1}^{-}(x),T_{l2}^{-}(x)\right],\max\left[T_{u1}^{-}(x),T_{u2}^{-}(x)\right]\right],\left[\min\left[I_{l1}^{-}(x),I_{l2}^{-}(x)\right],\min\left[I_{u1}^{-}(x),I_{u2}^{-}(x)\right]\right],\\ \left[\min\left[F_{l1}^{-}(x),F_{l2}^{-}(x)\right],\min\left[F_{u1}^{-}(x),F_{u2}^{-}(x)\right]\right]\end{array}\right) \text{ for all } x\in X.$$

The complement of $\overline{\overline{A}}_1$is denoted by $\overline{\overline{A}}_1^{C}$ is defined as

$T_{l1^c}^{+}(x)=1^{+}-T_{u1}^{+}(x),\quad T_{u1^c}^{+}(x)=1^{+}-T_{l1}^{+}(x), I_{l1^c}^{+}(x)=1^{+}-I_{u1}^{+}(x)$

$I_{u1^c}^{+}(x)=1^{+}-I_{l1}^{+}(x),\quad F_{l1^c}^{+}(x)=1^{+}-F_{u1}^{+}(x), F_{u1^c}^{+}(x)=1^{+}-F_{l1}^{+}(x)$, and

$T_{l1^c}^{-}(x)=1^{-}-T_{u1}^{-}(x),\quad T_{u1^c}^{-}(x)=1^{-}-T_{l1}^{-}(x), I_{l1^c}^{-}(x)=1^{-}-I_{u1}^{-}(x)$

$I_{u1^c}^{-}(x)=1^{-}-I_{l1}^{-}(x),\quad F_{l1^c}^{-}(x)=1^{-}-F_{u1}^{-}(x), F_{u1^c}^{-}(x)=1^{-}-F_{l1}^{-}(x)$ for all $x\in X$"

Definition 2.6. [(10)] **"**Let the two Interval valued bipolar neutrosophic numbers (IVBPNNs) are

$$\overline{\overline{A}}_1=\left\{\left[T_{l1}^{+}(x),T_{u1}^{+}(x)\right],\left[I_{l1}^{+}(x),I_{u1}^{+}(x)\right],\left[F_{l1}^{+}(x),F_{u1}^{+}(x)\right],\left[T_{l1}^{-}(x),T_{u1}^{-}(x)\right],\left[I_{l1}^{-}(x),I_{u1}^{-}(x)\right],\left[F_{l1}^{-}(x),F_{u1}^{-}(x)\right]\right\}$$

$$\overline{\overline{A}}_2=\left\{\left[T_{l2}^{+}(x),T_{u2}^{+}(x)\right],\left[I_{l2}^{+}(x),I_{u2}^{+}(x)\right],\left[F_{l2}^{+}(x),F_{u2}^{+}(x)\right],\left[T_{l2}^{-}(x),T_{u2}^{-}(x)\right],\left[I_{l2}^{-}(x),I_{u2}^{-}(x)\right],\left[F_{l2}^{-}(x),F_{u2}^{-}(x)\right]\right\}$$

Then the arithmetic operations on IVBPNNs are discussed below, where $\alpha>0$

1. $\alpha\overline{\overline{A_1}}=$

$$\left\{\begin{array}{c}\left[1-\left(1-T_{l1}^{+}\right)^{\alpha},1-\left(1-T_{u1}^{+}\right)^{\alpha}\right],\left[\left(I_{l1}^{+}\right)^{\alpha},\left(I_{u1}^{+}\right)^{\alpha}\right],\left[\left(F_{l1}^{+}\right)^{\alpha},\left(F_{u1}^{+}\right)^{\alpha}\right]\\ \left[-\left(-T_{l1}^{-}\right)^{\alpha},-\left(-T_{u1}^{-}\right)^{\alpha}\right],\left[-\left(-I_{l1}^{-}\right)^{\alpha},-\left(-I_{u1}^{-}\right)^{\alpha}\right],\left[-\left(1-\left(1-\left(-F_{l1}^{-}\right)\right)^{\alpha}\right),-\left(1-\left(1-\left(-F_{u1}^{-}\right)\right)^{\alpha}\right)\right]\end{array}\right\}$$

2. $\overline{\overline{A}}_1{}^{\alpha}=$

$$\left\{\begin{array}{c}\left[\left(T_{l}^{+}\right)^{\alpha},\left(T_{u}^{+}\right)^{\alpha}\right],\left[1-\left(1-I_{l}^{+}\right)^{\alpha},1-\left(1-I_{u}^{+}\right)^{\alpha}\right],\left[1-\left(1-F_{l}^{+}\right)^{\alpha},1-\left(1-F_{u}^{+}\right)^{\alpha}\right],\\ \left[-\left(1-\left(1-\left(-T_{l}^{-}\right)\right)^{\alpha}\right),-\left(1-\left(1-\left(-T_{u}^{-}\right)\right)^{\alpha}\right)\right],\left[-\left(-I_{l}^{-}\right)^{\alpha},-\left(-I_{u}^{-}\right)^{\alpha}\right],\left[-\left(-F_{l}^{-}\right)^{\alpha},-\left(-F_{u}^{-}\right)^{\alpha}\right]\end{array}\right\}$$

3. $$\overline{\overline{A}}_1+\overline{\overline{A}}_2=\left\{\begin{array}{c}\left[T_{l1}^{+}+T_{l2}^{+}-T_{l1}^{+}T_{l2}^{+},T_{u1}^{+}+T_{u2}^{+}-T_{u1}^{+}T_{u2}^{+}\right],\left[I_{l1}^{+}I_{l2}^{+},I_{u1}^{+}I_{u1}^{+}\right],\left[F_{l1}^{+}F_{l2}^{+},F_{u1}^{+}F_{u1}^{+}\right],\\ \left[-T_{l1}^{-}T_{l2}^{-},-T_{u1}^{-}T_{u2}^{-}\right],\left[-\left(-I_{l1}^{-}-I_{l2}^{-}-I_{l1}^{-}I_{l2}^{-}\right),-\left(-I_{u1}^{-}-I_{u2}^{-}-I_{u1}^{-}I_{u2}^{-}\right)\right]\\ \left[-\left(-F_{l1}^{-}-F_{l2}^{-}-F_{l1}^{-}F_{l2}^{-}\right),-\left(-F_{u1}^{-}-F_{u2}^{-}-F_{u1}^{-}F_{u2}^{-}\right)\right]\end{array}\right\}$$

4. $\overline{\overline{A}}_1\overline{\overline{A}}_2=$

$$\left\{\begin{array}{c}\left[T_{l1}^{+}T_{l2}^{+},T_{u1}^{+}T_{u2}^{+}\right],\left[I_{l1}^{+}+I_{l2}^{+}-I_{l1}^{+}I_{l2}^{+},I_{l1}^{+}+I_{u2}^{+}-I_{u1}^{+}I_{u2}^{+}\right],\left[F_{l1}^{+}+F_{l2}^{+}-F_{l1}^{+}F_{l2}^{+},F_{u1}^{+}+F_{u2}^{+}-F_{u1}^{+}F_{u2}^{+}\right],\\ \left[-\left(-T_{l1}^{-}-T_{l2}^{-}-T_{l1}^{-}T_{l2}^{-}\right),-\left(-T_{u1}^{-}-T_{u2}^{-}-T_{u1}^{-}T_{u2}^{-}\right)\right],\\ \left[I_{l1}^{-}I_{l2}^{-},I_{u1}^{-}I_{u2}^{-}\right],\left[F_{l1}^{-}F_{l2}^{-},F_{u1}^{-}F_{u2}^{-}\right]\end{array}\right\}"$$

3 Interval Valued Bipolar Triangular Neutrosophic Set [IVBPTrNS]

Here we discuss, IVBPTrNS and also its operations on its numbers:

Definition 3.1. An IVBPTrNS A in X is defined as an object of the form

$$A=\left\{\begin{array}{c}\left(\left(T_{l^1}^+(x),T_{l^2}^+(x),T_{l^3}^+(x)\right),\left(T_{u^1}^+(x),T_{u^2}^+(x),T_{u^3}^+(x)\right)\right],\\ \left(\left(I_{l^1}^+(x),I_{l^2}^+(x),I_{l^3}^+(x)\right),\left(I_{u^1}^+(x),I_{u^2}^+(x),I_{u^3}^+(x)\right)\right],\\ \left(\left(F_{l^1}^+(x),F_{l^2}^+(x),F_{l^3}^+(x)\right),\left(F_{u^1}^+(x),F_{u^2}^+(x),F_{u^3}^+(x)\right)\right]\\ \left(\left(T_{l^1}^-(x),T_{l^2}^-(x),T_{l^3}^-(x)\right),\left(T_{u^1}^-(x),T_{u^2}^-(x),T_{u^3}^-(x)\right)\right]\\ \left(\left(I_{l^1}^-(x),I_{l^2}^-(x),I_{l^3}^-(x)\right),\left(I_{u^1}^-(x),I_{u^2}^-(x),I_{u^3}^-(x)\right)\right]\\ \left(\left(F_{l^1}^-(x),F_{l^2}^-(x),F_{l^3}^-(x)\right),\left(F_{u^1}^-(x),F_{u^2}^-(x),F_{u^3}^-(x)\right)\right]\end{array}\right\}\text{ whereas}$$

$T_{l^1}^+,T_{l^2}^+,T_{l^3}^+,T_{u^1}^+,T_{u^2}^+,T_{u^3}^+,I_{l^1}^+,I_{l^2}^+,I_{l^3}^+,I_{u^1}^+,I_{u^2}^+,I_{u^3}^+,F_{l^1}^+,F_{l^2}^+,F_{l^3}^+,F_{u^1}^+,F_{u^2}^+,F_{u^3}^+ : X\to(0,1]$

$T_{l^1}^-,T_{l^2}^-,T_{l^3}^-,T_{u^1}^-,T_{u^2}^-,T_{u^3}^-,I_{l^1}^-,I_{l^2}^-,I_{l^3}^-,I_{u^1}^-,I_{u^2}^-,I_{u^3}^-,F_{l^1}^-,F_{l^2}^-,F_{l^3}^-,F_{u^1}^-,F_{u^2}^-,F_{u^3}^- : X\to(-1,0]$

An IVBPTrNN, A is represented by

$$A=\left\{\begin{array}{c}\left(\left(\underline{a}^+,\underline{b}^+,\underline{c}^+:T_l^+\right),\left(\left(\bar{a}^+,\bar{b}^+,\bar{c}^+:T_u^+\right)\right)\right],\left(\left(\underline{d}^+,\underline{e}^+,\underline{f}^+:I_l^+\right),\left(\bar{d}^+,\bar{e}^+,\bar{f}^+:I_u^+\right)\right],\\ \left[\left(\underline{p}^+,\underline{q}^+,\underline{r}^+:F_l^+\right),\left(\bar{p}^+,\bar{q}^+,\bar{r}^+:F_u^+\right)\right],\left[\left(\underline{a}^-,\underline{b}^-,\underline{c}^-:T_l^-\right),\left(\bar{a}^-,\bar{b}^-,\bar{c}^-:T_u^-\right)\right],\\ \left[\left(\underline{d}^-,\underline{e}^-,\underline{f}^-:I_l^-\right),\left(\bar{d}^-,\bar{e}^-,\bar{f}^-:I_u^-\right)\right],\left[\left(\underline{p}^-,\underline{q}^-,\underline{r}^-:F_l^-\right),\left(\bar{p}^-,\bar{q}^-,\bar{r}^-:F_u^-\right)\right]\end{array}\right\}$$

Definition 3.4. Let

$$A=\left\{\begin{array}{l}\left[\left(\underline{a}_1^+,\underline{b}_1^+,\underline{c}_1^+\right),\left(\bar{a}_1^+,\bar{b}_1^+,\bar{c}_1^+\right)\right],\left[\left(\underline{d}_1^+,\underline{e}_1^+,\underline{f}_1^+\right),\left(\bar{d}_1^+,\bar{e}_1^+,\bar{f}_1^+\right)\right],\\ \left[\left(\underline{p}_1^+,\underline{q}_1^+,\underline{r}_1^+\right),\left(\bar{p}_1^+,\bar{q}_1^+,\bar{r}_1^+\right)\right],\left[\left(\underline{a}_1^-,\underline{b}_1^-,\underline{c}_1^-\right),\left(\bar{a}_1^-,\bar{b}_1^-,\bar{c}_1^-\right)\right],\\ \left[\left(\underline{d}_1^-,\underline{e}_1^-,\underline{f}_1^-\right),\left(\bar{d}_1^-,\bar{e}_1^-,\bar{f}_1^-\right)\right],\left[\left(\underline{p}_1^-,\underline{q}_1^-,\underline{r}_1^-\right),\left(\bar{p}_1^-,\bar{q}_1^-,\bar{r}_1^-\right)\right]\end{array}\right\}\text{ and}$$

$$B=\left\{\begin{array}{l}\left[\left(\underline{a}_2^+,\underline{b}_2^+,\underline{c}_2^+\right),\left(\bar{a}_2^+,\bar{b}_2^+,\bar{c}_2^+\right)\right],\left[\left(\underline{d}_2^+,\underline{e}_2^+,\underline{f}_2^+\right),\left(\bar{d}_2^+,\bar{e}_2^+,\bar{f}_2^+\right)\right],\\ \left[\left(\underline{p}_2^+,\underline{q}_2^+,\underline{r}_2^+\right),\left(\bar{p}_2^+,\bar{q}_2^+,\bar{r}_2^+\right)\right],\left[\left(\underline{a}_2^-,\underline{b}_2^-,\underline{c}_2^-\right),\left(\bar{a}_2^-,\bar{b}_2^-,\bar{c}_2^-\right)\right],\\ \left[\left(\underline{d}_2^-,\underline{e}_2^-,\underline{f}_2^-\right),\left(\bar{d}_2^-,\bar{e}_2^-,\bar{f}_2^-\right)\right],\left[\left(\underline{p}_2^-,\underline{q}_2^-,\underline{r}_2^-\right),\left(\bar{p}_2^-,\bar{q}_2^-,\bar{r}_2^-\right)\right]\end{array}\right\}\text{ be two IVBPTrNNs}$$

1. $A\subseteq B$, iff

$$\begin{array}{llllll}\underline{a}_1^+\le\underline{a}_2^+, & \underline{b}_1^+\le\underline{b}_2^+, & \underline{c}_1^+\le\underline{c}_2^+, & \bar{a}_1^+\le\bar{a}_2^+, & \bar{b}_1^+\le\bar{b}_2^+, & \bar{c}_1^+\le\bar{c}_2^+\\ \underline{d}_1^+\ge\underline{d}_2^+, & \underline{e}_1^+\ge\underline{e}_2^+, & \underline{f}_1^+\ge\underline{f}_2^+, & \bar{d}_1^+\ge\bar{d}_2^+, & \bar{e}_1^+\ge\bar{e}_2^+, & \bar{f}_1^+\ge\bar{f}_2^+,\\ \underline{p}_1^+\ge\underline{p}_2^+, & \underline{q}_1^+\ge\underline{q}_2^+, & \underline{r}_1^+\ge\underline{r}_2^+, & \bar{p}_1^+\ge\bar{p}_2^+, & \bar{q}_1^+\ge\bar{q}_2^+, & \bar{r}_1^+\ge\bar{r}_2^+.\end{array}$$

and

$$\begin{array}{llllll}\underline{a}_1^-\le\underline{a}_2^-, & \underline{b}_1^-\le\underline{b}_2^-, & \underline{c}_1^-\le\underline{c}_2^-, & \bar{a}_1^-\le\bar{a}_2^-, & \bar{b}_1^-\le\bar{b}_2^-, & \bar{c}_1^-\le\bar{c}_2^-,\\ \underline{d}_1^-\ge\underline{d}_2^-, & \underline{e}_1^-\ge\underline{e}_2^-, & \underline{f}_1^-\ge\underline{f}_2^-, & \bar{d}_1^-\ge\bar{d}_2^-, & \bar{e}_1^-\ge\bar{e}_2^-, & \bar{f}_1^-\ge\bar{f}_2^-,\\ \underline{p}_1^-\ge\underline{p}_2^-, & \underline{q}_1^-\ge\underline{q}_2^-, & \underline{r}_1^-\ge\underline{r}_2^-, & \bar{p}_1^-\ge\bar{p}_2^-, & \bar{q}_1^-\ge\bar{q}_2^-, & \bar{r}_1^-\ge\bar{r}_2^-.\end{array}$$

2. A= B, iff

$$\begin{array}{llllll}\underline{a}_1^+=\underline{a}_2^+, & \underline{b}_1^+=\underline{b}_2^+, & \underline{c}_1^+=\underline{c}_2^+, & \bar{a}_1^+=\bar{a}_2^+, & \bar{b}_1^+=\bar{b}_2^+, & \bar{c}_1^+=\bar{c}_2^+,\\ \underline{d}_1^+=\underline{d}_2^+, & \underline{e}_1^+=\underline{e}_2^+, & \underline{f}_1^+=\underline{f}_2^+, & \bar{d}_1^+=\bar{d}_2^+, & \bar{e}_1^+=\bar{e}_2^+, & \bar{f}_1^+=\bar{f}_2^+,\\ \underline{p}_1^+=\underline{p}_2^+, & \underline{q}_1^+=\underline{q}_2^+, & \underline{r}_1^+=\underline{r}_2^+, & \bar{p}_1^+=\bar{p}_2^+, & \bar{q}_1^+=\bar{q}_2^+, & \bar{r}_1^+=\bar{r}_2^+.\end{array}$$

and

$$\begin{array}{llllll}\underline{a}_1^-=\underline{a}_2^-, & \underline{b}_1^-=\underline{b}_2^-, & \underline{c}_1^-=\underline{c}_2^-, & \bar{a}_1^-=\bar{a}_2^-, & \bar{b}_1^-=\bar{b}_2^-, & \bar{c}_1^-=\bar{c}_2^-,\\ \underline{d}_1^-=\underline{d}_2^-, & \underline{e}_1^-=\underline{e}_2^-, & \underline{f}_1^-=\underline{f}_2^-, & \bar{d}_1^-=\bar{d}_2^-, & \bar{e}_1^-=\bar{e}_2^-, & \bar{f}_1^-=\bar{f}_2^-,\\ \underline{p}_1^-=\underline{p}_2^-, & \underline{q}_1^-=\underline{q}_2^-, & \underline{r}_1^-=\underline{r}_2^-, & \bar{p}_1^-=\bar{p}_2^-, & \bar{q}_1^-=\bar{q}_2^-, & \bar{r}_1^-=\bar{r}_2^-.\end{array}$$

3. The union of A and B is defined as

$(A\cup B)(x)=$

$$\left[\left(\max\left\{\underline{a}_1^+,\underline{a}_2^+\right\},\max\left\{\underline{b}_1^+,\underline{b}_2^+\right\},\max\left\{\underline{c}_1^+,\underline{c}_2^+\right\}\right),\left(\max\left\{\bar{a}_1^+,\bar{a}_2^+\right\},\max\left\{\bar{b}_1^+,\bar{b}_2^+\right\},\max\left\{\bar{c}_1^+,\bar{c}_2^+\right\}\right)\right],$$
$$\left[\left(\min\left\{\underline{d}_1^+,\underline{d}_2^+\right\},\min\left\{\underline{e}_1^+,\underline{e}_2^+\right\},\min\left\{\underline{f}_1^+,\underline{f}_2^+\right\}\right),\left(\min\left\{\bar{d}_1^+,\bar{d}_2^+\right\},\min\left\{\bar{e}_1^+,\bar{e}_2^+\right\},\min\left\{\bar{f}_1^+,\bar{f}_2^+\right\}\right)\right],$$
$$\left[\left(\min\left\{\underline{p}_1^+,\underline{p}_2^+\right\},\min\left\{\underline{q}_1^+,\underline{q}_2^+\right\},\min\left\{\underline{r}_1^+,\underline{r}_2^+\right\}\right),\left(\min\left\{\bar{p}_1^+,\bar{p}_2^+\right\},\min\left\{\bar{q}_1^+,\bar{q}_2^+\right\},\min\left\{\bar{r}_1^+,\bar{r}_2^+\right\}\right)\right],$$
$$\left[\left(\min\left\{\underline{a}_1^-,\underline{a}_2^-\right\},\min\left\{\underline{b}_1^-,\underline{b}_2^-\right\},\min\left\{\underline{c}_1^-,\underline{c}_2^-\right\}\right),\left(\min\left\{\bar{a}_1^-,\bar{a}_2^-\right\},\min\left\{\bar{b}_1^-,\bar{b}_2^-\right\},\min\left\{\bar{c}_1^-,\bar{c}_2^-\right\}\right)\right],$$
$$\left[\left(\max\left\{\underline{d}_1^-,\underline{d}_2^-\right\},\max\left\{\underline{e}_1^-,\underline{e}_2^-\right\},\max\left\{\underline{f}_1^-,\underline{f}_2^-\right\}\right),\left(\max\left\{\bar{d}_1^-,\bar{d}_2^-\right\},\max\left\{\bar{e}_1^-,\bar{e}_2^-\right\},\max\left\{\bar{f}_1^-,\bar{f}_2^-\right\}\right)\right],$$
$$\left[\left(\max\left\{\underline{p}_1^-,\underline{p}_2^-\right\},\max\left\{\underline{q}_1^-,\underline{q}_2^-\right\},\max\left\{\underline{r}_1^-,\underline{r}_2^-\right\}\right),\left(\max\left\{\bar{p}_1^-,\bar{p}_2^-\right\},\max\left\{\bar{q}_1^-,\bar{q}_2^-\right\},\max\left\{\bar{r}_1^-,\bar{r}_2^-\right\}\right)\right]$$

4. The intersection of A and B is defined as
$(A\cap B)(x)=$

$$\left[\left(\min\left\{\underline{a}_1^+,\underline{a}_2^+\right\},\min\left\{\underline{b}_1^+,\underline{b}_2^+\right\},\min\left\{\underline{c}_1^+,\underline{c}_2^+\right\}\right),\left(\min\left\{\bar{a}_1^+,\bar{a}_2^+\right\},\min\left\{\bar{b}_1^+,\bar{b}_2^+\right\},\min\left\{\bar{c}_1^+,\bar{c}_2^+\right\}\right)\right],$$
$$\left[\left(\max\left\{\underline{d}_1^+,\underline{d}_2^+\right\},\max\left\{\underline{e}_1^+,\underline{e}_2^+\right\},\max\left\{\underline{f}_1^+,\underline{f}_2^+\right\}\right),\left(\max\left\{\bar{d}_1^+,\bar{d}_2^+\right\},\max\left\{\bar{e}_1^+,\bar{e}_2^+\right\},\max\left\{\bar{f}_1^+,\bar{f}_2^+\right\}\right)\right],$$
$$\left[\left(\max\left\{\underline{p}_1^+,\underline{p}_2^+\right\},\max\left\{\underline{q}_1^+,\underline{q}_2^+\right\},\max\left\{\underline{r}_1^+,\underline{r}_2^+\right\}\right),\left(\max\left\{\bar{p}_1^+,\bar{p}_2^+\right\},\max\left\{\bar{q}_1^+,\bar{q}_2^+\right\},\max\left\{\bar{r}_1^+,\bar{r}_2^+\right\}\right)\right],$$
$$\left[\left(\max\left\{\underline{a}_1^-,\underline{a}_2^-\right\},\max\left\{\underline{b}_1^-,\underline{b}_2^-\right\},\max\left\{\underline{c}_1^-,\underline{c}_2^-\right\}\right),\left(\max\left\{\bar{a}_1^-,\bar{a}_2^-\right\},\max\left\{\bar{b}_1^-,\bar{b}_2^-\right\},\max\left\{\bar{c}_1^-,\bar{c}_2^-\right\}\right)\right],$$
$$\left[\left(\min\left\{\underline{d}_1^-,\underline{d}_2^-\right\},\min\left\{\underline{e}_1^-,\underline{e}_2^-\right\},\min\left\{\underline{f}_1^-,\underline{f}_2^-\right\}\right),\left(\min\left\{\bar{d}_1^-,\bar{d}_2^-\right\},\min\left\{\bar{e}_1^-,\bar{e}_2^-\right\},\min\left\{\bar{f}_1^-,\bar{f}_2^-\right\}\right)\right],$$
$$\left[\left(\min\left\{\underline{p}_1^-,\underline{p}_2^-\right\},\min\left\{\underline{q}_1^-,\underline{q}_2^-\right\},\min\left\{\underline{r}_1^-,\underline{r}_2^-\right\}\right),\left(\min\left\{\bar{p}_1^-,\bar{p}_2^-\right\},\min\left\{\bar{q}_1^-,\bar{q}_2^-\right\},\min\left\{\bar{r}_1^-,\bar{r}_2^-\right\}\right)\right].$$

5. The complement of A is denoted by A^C is defined as,

$$\underline{a}_1^{+C}=1^+-\bar{a}_1^+,\quad \bar{a}_1^{+C}=1^+-\underline{a}_1^+,\quad \underline{b}_1^{+C}=1^+-\bar{b}_1^+,\quad \bar{b}_1^{+C}=1^+-\underline{b}_1^+,$$
$$\underline{c}_1^{+C}=1^+-\bar{c}_1^+,\quad \bar{c}_1^{+C}=1^+-\underline{c}_1^+,\quad \underline{d}_1^{+C}=1^+-\bar{d}_1^+,\quad \bar{d}_1^{+C}=1^+-\underline{d}_1^+,$$
$$\underline{e}_1^{+C}=1^+-\bar{e}_1^+,\quad \bar{e}_1^{+C}=1^+-\underline{e}_1^+,\quad \underline{f}_1^{+C}=1^+-\bar{f}_1^+,\quad \bar{f}_1^{+C}=1^+-\underline{f}_1^+,$$
$$\underline{p}_1^{+C}=1^+-\bar{p}_1^+,\quad \bar{p}_1^{+C}=1^+-\underline{p}_1^+,\quad \underline{q}_1^{+C}=1^+-\bar{q}_1^+,\quad \bar{q}_1^{+C}=1^+-\underline{q}_1^+,$$
$$\underline{r}_1^{+C}=1^+-\bar{r}_1^+,\quad \bar{r}_1^{+C}=1^+-\underline{r}_1^+$$

and

$$\underline{a}_1^{-C}=1^--\bar{a}_1^-,\quad \bar{a}_1^{-C}=1^--\underline{a}_1^-,\quad \underline{b}_1^{-C}=1^--\bar{b}_1^-,\quad \bar{b}_1^{-C}=1^--\underline{b}_1^-,$$
$$\underline{c}_1^{-C}=1^--\bar{c}_1^-,\quad \bar{c}_1^{-C}=1^--\underline{c}_1^-,\quad \underline{d}_1^{-C}=1^--\bar{d}_1^-,\quad \bar{d}_1^{-C}=1^--\underline{d}_1^-,$$
$$\underline{e}_1^{-C}=1^--\bar{e}_1^-,\quad \bar{e}_1^{-C}=1^--\underline{e}_1^-,\quad \underline{f}_1^{-C}=1^--\bar{f}_1^-,\quad \bar{f}_1^{-C}=1^--\underline{f}_1^-,$$
$$\underline{p}_1^{-C}=1^--\bar{p}_1^-,\quad \bar{p}_1^{-C}=1^--\underline{p}_1^-,\quad \underline{q}_1^{-C}=1^--\bar{q}_1^-,\quad \bar{q}_1^{-C}=1^--\underline{q}_1^-,$$
$$\underline{r}_1^{-C}=1^--\bar{r}_1^-,\quad \bar{r}_1^{-C}=1^--\underline{r}_1^-.$$

Definition 3.3. Let A and B be two IVBPTrNNs,

$$A=\left\{\begin{array}{l}\left(\left(\underline{a}_1^+,\underline{b}_1^+,\underline{c}_1^+\right),\left(\bar{a}_1^+,\bar{b}_1^+,\bar{c}_1^+\right)\right],\left(\left(\underline{d}_1^+,\underline{e}_1^+,\underline{f}_1^+\right),\left(\bar{d}_1^+,\bar{e}_1^+,\bar{f}_1^+\right)\right],\\ \left(\left(\underline{p}_1^+,\underline{q}_1^+,\underline{r}_1^+\right),\left(\bar{p}_1^+,\bar{q}_1^+,\bar{r}_1^+\right)\right],\left(\left(\underline{a}_1^-,\underline{b}_1^-,\underline{c}_1^-\right),\left(\bar{a}_1^-,\bar{b}_1^-,\bar{c}_1^-\right)\right],\\ \left(\left(\underline{d}_1^-,\underline{e}_1^-,\underline{f}_1^-\right),\left(\bar{d}_1^-,\bar{e}_1^-,\bar{f}_1^-\right)\right],\left(\left(\underline{p}_1^-,\underline{q}_1^-,\underline{r}_1^-\right),\left(\bar{p}_1^-,\bar{q}_1^-,\bar{r}_1^-\right)\right]\end{array}\right\}\text{ and}$$

$$B=\left\{\begin{array}{l}\left(\left(\underline{a}_2^+,\underline{b}_2^+,\underline{c}_2^+\right),\left(\bar{a}_2^+,\bar{b}_2^+,\bar{c}_2^+\right)\right],\left(\left(\underline{d}_2^+,\underline{e}_2^+,\underline{f}_2^+\right),\left(\bar{d}_2^+,\bar{e}_2^+,\bar{f}_2^+\right)\right],\\ \left(\left(\underline{p}_2^+,\underline{q}_2^+,\underline{r}_2^+\right),\left(\bar{p}_2^+,\bar{q}_2^+,\bar{r}_2^+\right)\right],\left(\left(\underline{a}_2^-,\underline{b}_2^-,\underline{c}_2^-\right),\left(\bar{a}_2^-,\bar{b}_2^-,\bar{c}_2^-\right)\right],\\ \left(\left(\underline{d}_2^-,\underline{e}_2^-,\underline{f}_2^-\right),\left(\bar{d}_2^-,\bar{e}_2^-,\bar{f}_2^-\right)\right],\left(\left(\underline{p}_2^-,\underline{q}_2^-,\underline{r}_2^-\right),\left(\bar{p}_2^-,\bar{q}_2^-,\bar{r}_2^-\right)\right]\end{array}\right\}$$

Then the operations are defined as below, where $\alpha > 0$

1. $\alpha A =$

$$\left\{\begin{array}{c}
\left[\left(1-(1-\underline{a}_1^+)^\alpha, 1-(1-\underline{b}_1^+)^\alpha, 1-(1-\underline{c}_1^+)^\alpha\right),\left(1-(1-\bar{a}_1^+)^\alpha, 1-(1-\bar{b}_1^+)^\alpha, (1-(1-\bar{c}_1^+)^\alpha)\right)\right],\\
\left[\left((\underline{d}_1^+)^\alpha, (\underline{e}_1^+)^\alpha, (\underline{f}_1^+)^\alpha\right),\left((\bar{d}_1^+)^\alpha, (\bar{e}_1^+)^\alpha, (\bar{f}_1^+)^\alpha\right)\right],\left[\left((\underline{p}_1^+)^\alpha, (\underline{q}_1^+)^\alpha, (\underline{r}_1^+)^\alpha\right),\left((\bar{p}_1^+)^\alpha, (\bar{q}_1^+)^\alpha, (\bar{r}_1^+)^\alpha\right)\right],\\
\left[\left(-(-\underline{a}_1^-)^\alpha, -(-\underline{b}_1^-)^\alpha, -(-\underline{c}_1^-)^\alpha\right),\left(-(-\bar{a}_1^-)^\alpha, -(-\bar{b}_1^-)^\alpha, -(-\bar{c}_1^-)^\alpha\right)\right],\\
\left[\left(-(-\underline{d}_1^-)^\alpha, -(-\underline{e}_1^-)^\alpha, -(-\underline{f}_1^-)^\alpha\right),\left(-(-\bar{d}_1^-)^\alpha, -(-\bar{e}_1^-)^\alpha, -(-\bar{f}_1^-)^\alpha\right)\right],\\
[\left(-\left(1-\left(1-(-\underline{p}_1^-)\right)^\alpha\right), -\left(1-\left(1-(-\underline{q}_1^-)\right)^\alpha\right), -\left(1-\left(1-(-\underline{r}_1^-)\right)^\alpha\right)\right),\\
\left(-\left(1-(1-(-\bar{p}_1^-))^\alpha\right), -\left(1-(1-(-\bar{q}_1^-))^\alpha\right), -\left(1-(1-(-\bar{r}_1^-))^\alpha\right)\right)]
\end{array}\right\}$$

2. $A^\alpha =$

$$\left\{\begin{array}{c}
\left[\left((\underline{a}_1^+)^\alpha, (\underline{b}_1^+)^\alpha, (\underline{c}_1^+)^\alpha\right),\left((\bar{a}_1^+)^\alpha, (\bar{b}_1^+)^\alpha, (\bar{c}_1^+)^\alpha\right)\right],\\
\left[\left(1-(1-\underline{d}_1^+)^\alpha, 1-(1-\underline{e}_1^+)^\alpha, 1-(1-\underline{f}_1^+)^\alpha\right),\left(1-(1-\bar{d}_1^+)^\alpha, 1-(1-\bar{e}_1^+)^\alpha, (1-(1-\bar{f}_1^+)^\alpha)\right)\right],\\
\left[\left(1-(1-\underline{p}_1^+)^\alpha, 1-(1-\underline{q}_1^+)^\alpha, 1-(1-\underline{r}_1^+)^\alpha\right),\left(1-(1-\bar{p}_1^+)^\alpha, 1-(1-\bar{q}_1^+)^\alpha, (1-(1-\bar{r}_1^+)^\alpha)\right)\right],\\
[\left(-\left(1-\left(1-(-\underline{a}_1^-)\right)^\alpha\right), -\left(1-\left(1-(-\underline{b}_1^-)\right)^\alpha\right), -\left(1-\left(1-(-\underline{c}_1^-)\right)^\alpha\right)\right),\\
\left(-\left(1-(1-(-\bar{a}_1^-))^\alpha\right), -\left(1-\left(1-(-\bar{b}_1^-)\right)^\alpha\right), -\left(1-(1-(-\bar{c}_1^-))^\alpha\right)\right)]\\
\left[\left(-(-\underline{d}_1^-)^\alpha, -(-\underline{e}_1^-)^\alpha, -(-\underline{f}_1^-)^\alpha\right),\left(-(-\bar{d}_1^-)^\alpha, -(-\bar{e}_1^-)^\alpha, -(-\bar{f}_1^-)^\alpha\right)\right],\\
\left[\left(-(-\underline{p}_1^-)^\alpha, -(-\underline{q}_1^-)^\alpha, -(-\underline{r}_1^-)^\alpha\right),\left(-(-\bar{p}_1^-)^\alpha, -(-\bar{q}_1^-)^\alpha, -(-\bar{r}_1^-)^\alpha\right)\right]
\end{array}\right\}$$

3.

$$A+B=\left\{\begin{array}{c}
[\left(\underline{a}_1^+ + \underline{a}_2^+ - \underline{a}_1^+.\underline{a}_2^+, \underline{b}_1^+ + \underline{b}_2^+ - \underline{b}_1^+.\underline{b}_2^+, \underline{c}_1^+ + \underline{c}_2^+ - \underline{c}_1^+.\underline{c}_2^+\right),\\
\left(\bar{a}_1^+ + \bar{a}_2^+ - \bar{a}_1^+.\bar{a}_2^+, \bar{b}_1^+ + \bar{b}_2^+ - \bar{b}_1^+.\bar{b}_2^+, \bar{c}_1^+ + \bar{c}_2^+ - \bar{c}_1^+.\bar{c}_2^+\right)],\\
\left[\left(\underline{d}_1^+\underline{d}_2^+, \underline{e}_1^+\underline{e}_2^+, \underline{f}_1^+\underline{f}_2^+\right),\left(\bar{d}_1^+\bar{d}_2^+, \bar{e}_1^+\bar{e}_2^+, \bar{f}_1^+\bar{f}_2^+\right)\right],\\
\left[\left(\underline{p}_1^+\underline{p}_2^+, \underline{q}_1^+\underline{q}_2^+, \underline{r}_1^+\underline{r}_2^+\right),\left(\bar{p}_1^+\bar{p}_2^+, \bar{q}_1^+\bar{q}_2^+, \bar{r}_1^+\bar{r}_2^+\right)\right],\\
\left[\left(-\underline{a}_1^-\underline{a}_2^-, -\underline{b}_1^-\underline{b}_2^-, \underline{c}_1^-\underline{c}_2^-\right),\left(-\bar{a}_1^-\bar{a}_2^-, -\bar{b}_1^-\bar{b}_2^-, -\bar{c}_1^-\bar{c}_2^-\right)\right],\\
[\left(-\left(-\underline{d}_1^- \underline{-d}_2^- - \underline{d}_1^-\underline{d}_2^-\right), -\left(-\underline{e}_1^- - \underline{e}_2^- - \underline{e}_1^-\underline{e}_2^-\right), -\left(-\underline{f}_1^- - \underline{f}_2^- - \underline{f}_1^-\underline{f}_2^-\right)\right),\\
\left(-\left(\overline{-d}_1^- \overline{-d}_2^- - \bar{d}_1^-\bar{d}_2^-\right), -\left(-\bar{e}_1^- \overline{-e}_2^- - \bar{e}_1^-\bar{e}_2^-\right), -\left(\overline{-f}_1^- \overline{-f}_2^- - \bar{f}_1^-\bar{f}_2^-\right)\right)],\\
[\left(-\left(-\underline{p}_1^- \underline{-p}_2^- - \underline{p}_1^-\underline{p}_2^-\right), -\left(-\underline{q}_1^- - \underline{q}_2^- - \underline{q}_1^-\underline{q}_2^-\right), -\left(-\underline{r}_1^- {}_- \underline{r}_2^- - \underline{r}_1^-\underline{r}_2^-\right)\right),\\
\left(-\left(\overline{-p}_1^- - \bar{p}_2^- - \bar{p}_1^-\bar{p}_2^-\right), -\left(\overline{-q}_1^- {}_- \bar{q}_2^- - \bar{q}_1^-\bar{q}_2^-\right), -\left(-\bar{r}_1^- - \bar{r}_2^- - \bar{r}_1^-\bar{r}_2^-\right)\right)]
\end{array}\right\}$$

4.

$$AB=\left\{\begin{array}{c}
\left[\underline{a}_1^+\underline{a}_2^+, \underline{b}_1^+\underline{b}_2^+, \underline{c}_1^+\underline{c}_2^+, \bar{a}_1^+\bar{a}_2^+, \bar{b}_1^+\bar{b}_2^+, \bar{c}_1^+\bar{c}_2^+\right]\\
[\left(\underline{d}_1^+ + \underline{d}_2^+ - \underline{d}_1^+.\underline{d}_2^+, \underline{e}_1^+ + \underline{e}_2^+ - \underline{e}_1^+.\underline{e}_2^+, \underline{f}_1^+ + \underline{f}_2^+ - \underline{f}_1^+.\underline{f}_2^+\right),\\
\left(\bar{d}_1^+ + \bar{d}_2^+ - \bar{d}_1^+.\bar{d}_2^+, \bar{e}_1^+ + \bar{e}_2^+ - \bar{e}_1^+.\bar{e}_2^+, \bar{f}_1^+ + \bar{f}_2^+ - \bar{f}_1^+.\bar{f}_2^+\right)],\\
[\left(\underline{p}_1^+ + \underline{p}_2^+ - \underline{p}_1^+.\underline{p}_2^+, \underline{q}_1^+ + q_2^+ - \underline{q}_1^+.\underline{q}_2^+, \underline{r}_1^+ + \underline{r}_2^+ - \underline{r}_1^+.\underline{r}_2^+\right),\\
\left(\bar{p}_1^+ + \bar{p}_2^+ - \bar{p}_1^+.\bar{p}_2^+, \bar{q}_1^+ + \bar{q}_2^+ - \bar{q}_1^+.\bar{q}_2^+, \bar{r}_1^+ + \bar{r}_2^+ - \bar{r}_1^+.\bar{r}_2^+\right)],\\
[-(-\underline{a}_1^- - \underline{a}_2^- - \underline{a}_1^-.\underline{a}_2^-, -b_1^- - \underline{b}_2^- - \underline{b}_1^-.\underline{b}_2^-, -\underline{c}_1^- - \underline{c}_2^- - \underline{c}_1^-.\underline{c}_2^-),\\
-(-\bar{a}_1^- - \bar{a}_2^- - \bar{a}_1^-.\bar{a}_2^-, -\bar{b}_1^- - \bar{b}_2^- - \bar{b}_1^-.\bar{b}_2^-, -\bar{c}_1^- - \bar{c}_2^- - \bar{c}_1^-.\bar{c}_2^-)],\\
[(\underline{d}_1^-\underline{d}_2^-, \underline{e}_1^-\underline{e}_2^-, \underline{f}_1^-\underline{f}_2^-), (\bar{d}_1^-\bar{d}_2^-, \bar{e}_1^-\bar{e}_2^-, \bar{f}_1^-\bar{f}_2^-)],\\
[(\underline{p}_1^-\underline{p}_2^-, \underline{q}_1^-\underline{q}_2^-, \underline{r}_1^-\underline{r}_2^-), (\bar{p}_1^-\bar{p}_2^-, \bar{q}_1^-\bar{q}_2^-, \bar{r}_1^-\bar{r}_2^-)]
\end{array}\right\}$$

Definition 3.4 . Let

$$A_k = \left\{\begin{array}{l}
\left[\left(\underline{a}_k^+, \underline{b}_k^+, c_k^+\right), \left(\bar{a}_k^+, \bar{b}_k^+, \bar{c}_k^+\right)\right], \left[\left(\underline{d}_k^+, e_k^+, \underline{f}_k^+\right), \left(\bar{d}_k^+, \bar{e}_k^+, \bar{f}_k^+\right)\right],\\
\left[\left(\underline{p}_k^+, \underline{q}_k^+, \underline{r}_k^+\right), \left(\bar{p}_k^+, \bar{q}_k^+, \bar{r}_k^+\right)\right], \left[\left(\underline{a}_k^-, \underline{b}_k^-, \underline{c}_k^-\right), \left(\bar{a}_k^-, \bar{b}_k^-, \bar{c}_k^-\right)\right],\\
\left[\left(\underline{d}_k^-, \underline{e}_k^-, \underline{f}_k^-\right), \left(\bar{d}_k^-, \bar{e}_k^-, \bar{f}_k^-\right)\right], \left[\left(\underline{p}_k^-, \underline{q}_k^-, \underline{r}_k^-\right), \left(\bar{p}_k^-, \bar{q}_k^-, \bar{r}_k^-\right)\right]
\end{array}\right\}, k = 1, 2, \ldots, n$$ be a

family of interval valued bipolar triangular neutrosophic numbers, A mapping $A_W : \xi_n \to \xi$ is called interval valued bipolar neutrosophic weighted average operator if it satisfies,$A_W\left(A_1, A_2, \ldots, A_n\right) = \sum_{k=1}^{n} w_k A_k$

$$\left\{\begin{array}{c} \%stack\left[\begin{array}{c}\left(1-\prod_{k=1}^{n}\left(1-\underline{a}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\underline{b}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\underline{c}_k^{+}\right)^{w_k}\right), \\ \left(1-\prod_{k=1}^{n}\left(1-\bar{a}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\bar{b}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\bar{c}_k^{+}\right)^{w_k}\right)\end{array}\right] \\ \left[\left(\prod_{k=1}^{n}\left(\underline{d}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\underline{e}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\underline{f}_k^{+}\right)^{w_k}\right), \left(\prod_{k=1}^{n}\left(\bar{d}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\bar{e}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\bar{f}_k^{+}\right)^{w_k}\right)\right] \% \\ \left[\left(\prod_{k=1}^{n}\left(\underline{p}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\underline{q}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\underline{r}_k^{+}\right)^{w_k}\right), \left(\prod_{k=1}^{n}\left(\bar{p}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\bar{q}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\bar{r}_k^{+}\right)^{w_k}\right)\right], \\ \left[\left(-\prod_{k=1}^{n}\left(-\underline{a}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\underline{b}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\underline{c}_k^{-}\right)^{w_k}\right), \left(-\prod_{k=1}^{n}\left(-\bar{a}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\bar{b}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\bar{c}_k^{-}\right)^{w_k}\right)\right], \\ \left[\begin{array}{c}\left(-\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{d}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{e}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{f}_k^{-}\right)\right)^{w_k}\right)\right), \\ \left(-\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{d}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{e}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{f}_k^{-}\right)\right)^{w_k}\right)\right)\end{array}\right] \\ \left[\begin{array}{c}\left(-\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{p}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{q}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{r}_k^{-}\right)\right)^{w_k}\right)\right), \\ \left(-\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{p}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{q}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{r}_k^{-}\right)\right)^{w_k}\right)\right)\end{array}\right]\end{array}\right\}$$

w_k is the weight of $A_k (k = 1, 2, \ldots, n)$, $w_k \in (0, 1]$ and also $\sum_{k=1}^{n} w_k = 1$.

Definition 3.5 . Let $A_k = \left\{\begin{array}{c}\left[\left(\underline{a}_k^{+}, \underline{b}_k^{+}, \underline{c}_k^{+}\right), \left(\bar{a}_k^{+}, \bar{b}_k^{+}, \bar{c}_k^{+}\right)\right], \left[\left(\underline{d}_k^{+}, \underline{e}_k^{+}, \underline{f}_k^{+}\right), \left(\bar{d}_k^{+}, \bar{e}_k^{+}, \bar{f}_k^{+}\right)\right], \\ \left[\left(\underline{p}_k^{+}, \underline{q}_k^{+}, \underline{r}_k^{+}\right), \left(\bar{p}_k^{+}, \bar{q}_k^{+}, \bar{r}_k^{+}\right)\right], \left[\left(\underline{a}_k^{-}, \underline{b}_k^{-}, \underline{c}_k^{-}\right), \left(\bar{a}_k^{-}, \bar{b}_k^{-}, \bar{c}_k^{-}\right)\right] \\ \left[\left(\underline{d}_k^{-}, \underline{e}_k^{-}, \underline{f}_k^{-}\right), \left(\bar{d}_k^{-}, \bar{e}_k^{-}, \bar{f}_k^{-}\right)\right], \left[\left(\underline{p}_k^{-}, \underline{q}_k^{-}, \underline{r}_k^{-}\right), \left(\bar{p}_k^{-}, \bar{q}_k^{-}, \bar{r}_k^{-}\right)\right]\end{array}\right\}$, $k = 1, 2, \ldots, n$ be a family of interval valued bipolar triangular neutrosophic numbers, A mapping $G_W : \xi_n \to \xi$ is called interval valued bipolar neutrosophic weighted average operator if it satisfies, $G_W\left(A_1, A_2, \ldots, A_n\right) = \sum_{k=1}^{n} {A_k}^{w_k}$.

$$= \left\{\begin{array}{c} \left[\left(\prod_{k=1}^{n}\left(\underline{a}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\underline{b}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\underline{c}_k^{+}\right)^{w_k}\right), \left(\prod_{k=1}^{n}\left(\bar{a}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\bar{b}_k^{+}\right)^{w_k}, \prod_{k=1}^{n}\left(\bar{c}_k^{+}\right)^{w_k}\right)\right] \\ \left[\begin{array}{c}\left(1-\prod_{k=1}^{n}\left(1-\underline{d}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\underline{e}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\underline{f}_k^{+}\right)^{w_k}\right), \\ \left(1-\prod_{k=1}^{n}\left(1-\bar{d}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\bar{e}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\bar{f}_k^{+}\right)^{w_k}\right)\end{array}\right] \\ \left[\begin{array}{c}\left(1-\prod_{k=1}^{n}\left(1-\underline{p}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\underline{q}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\underline{r}_k^{+}\right)^{w_k}\right), \\ \left(1-\prod_{k=1}^{n}\left(1-\bar{p}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\bar{q}_k^{+}\right)^{w_k}, 1-\prod_{k=1}^{n}\left(1-\bar{r}_k^{+}\right)^{w_k}\right)\end{array}\right] \\ \left[\begin{array}{c}\left(-\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{a}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{b}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\underline{c}_k^{-}\right)\right)^{w_k}\right)\right), \\ \left(-\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{a}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{b}_k^{-}\right)\right)^{w_k}\right), -\left(1-\prod_{k=1}^{n}\left(1-\left(-\bar{c}_k^{-}\right)\right)^{w_k}\right)\right)\end{array}\right] \\ \left[\left(-\prod_{k=1}^{n}\left(-\underline{d}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\underline{e}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\underline{f}_k^{-}\right)^{w_k}\right), \left(-\prod_{k=1}^{n}\left(-\bar{d}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\bar{e}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\bar{f}_k^{-}\right)^{w_k}\right)\right] \\ \left[\left(-\prod_{k=1}^{n}\left(-\underline{p}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\underline{q}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\underline{r}_k^{-}\right)^{w_k}\right), \left(-\prod_{k=1}^{n}\left(-\bar{p}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\bar{q}_k^{-}\right)^{w_k}, -\prod_{k=1}^{n}\left(-\bar{r}_k^{-}\right)^{w_k}\right)\right]\end{array}\right\}$$

w_k is the weight of $A_k (k = 1, 2, \ldots, n)$, $w_k \in (0, 1]$ and also $\sum_{k=1}^{n} w_k = 1$.

4 Score, Accuracy and Certainty functions

Definition 4.1 Let

$\hat{a} = \left\{\begin{array}{c}\left[\left(\underline{a}^{+}, \underline{b}^{+}, \underline{c}^{+}\right), \left(\bar{a}^{+}, \bar{b}^{+}, \bar{c}^{+}\right)\right], \left[\left(\underline{d}^{+}, \underline{e}^{+}, \underline{f}^{+}\right), \left(\bar{d}^{+}, \bar{e}^{+}, \bar{f}^{+}\right)\right], \\ \left[\left(\underline{p}^{+}, \underline{q}^{+}, \underline{r}^{+}\right), \left(\bar{p}^{+}, \bar{q}^{+}, \bar{r}^{+}\right)\right], \left[\left(\underline{a}^{-}, \underline{b}^{-}, \underline{c}^{-}\right), \left(\bar{a}^{-}, \bar{b}^{-}, \bar{c}^{-}\right)\right], \\ \left[\left(\underline{d}^{-}, \underline{e}^{-}, \underline{f}^{-}\right), \left(\bar{d}^{-}, \bar{e}^{-}, \bar{f}^{-}\right)\right], \left[\left(\underline{p}^{-}, \underline{q}^{-}, \underline{r}^{-}\right), \left(\bar{p}^{-}, \bar{q}^{-}, \bar{r}^{-}\right)\right]\end{array}\right\}$ be ar be an interval valued bipolar triangular neutrosophic number.

The Score function is defined as,

$$S(\hat{a})=\frac{1}{12}\left[6+\frac{\underline{a}^{+}+\underline{b}^{+}+\underline{c}^{+}}{3}+\frac{\bar{a}^{+}+\bar{b}^{+}+\bar{c}^{+}}{3}-\frac{\underline{d}^{+}+\underline{e}^{+}+\underline{f}^{+}}{3}-\frac{\bar{d}^{+}+\bar{e}^{+}+\bar{f}^{+}}{3}\right.$$
$$-\frac{\underline{p}^{+}+\underline{q}^{+}+\underline{r}^{+}}{3}-\frac{\bar{p}^{+}+\bar{q}^{+}+\bar{r}^{+}}{3}+\frac{\underline{a}^{-}+\underline{b}^{-}+\underline{c}^{-}}{3}+\frac{\bar{a}^{-}+\bar{b}^{-}+\bar{c}^{-}}{3}$$
$$\left.-\frac{\underline{d}^{-}+\underline{e}^{-}+\underline{f}^{-}}{3}-\frac{\bar{d}^{-}+\bar{e}^{-}+\bar{f}^{-}}{3}-\frac{\underline{p}^{-}+\underline{q}^{-}+\underline{r}^{-}}{3}-\frac{\bar{p}^{-}+\bar{q}^{-}+\bar{r}^{-}}{3}\right]$$

The Accuracy function is defined as,

$$A(\hat{a})=\frac{\underline{a}^{+}+\underline{b}^{+}+\underline{c}^{+}}{3}+\frac{\bar{a}^{+}+\bar{b}^{+}+\bar{c}^{+}}{3}-\frac{\underline{p}^{+}+\underline{q}^{+}+\underline{r}^{+}}{3}-\frac{\bar{p}^{+}+\bar{q}^{+}+\bar{r}^{+}}{3}+\frac{\underline{a}^{-}+\underline{b}^{-}+\underline{c}^{-}}{3}+\frac{\bar{a}^{-}+\bar{b}^{-}+\bar{c}^{-}}{3}-\frac{\underline{p}^{-}+\underline{q}^{-}+\underline{r}^{-}}{3}-\frac{\bar{p}^{-}+\bar{q}^{-}+\bar{r}^{-}}{3}.$$

The certainty function is defined as, $C(\hat{a})=\frac{\underline{a}^{+}+\underline{b}^{+}+\underline{c}^{+}}{3}+\frac{\bar{a}^{+}+\bar{b}^{+}+\bar{c}^{+}}{3}-\frac{\underline{p}^{-}+\underline{q}^{-}+\underline{r}^{-}}{3}-\frac{\bar{p}^{-}+\bar{q}^{-}+\bar{r}^{-}}{3}$.

5 Introduction of MCDM problem in Interval valued bipolar triangular neutrosophic environment

The best school has to be selected among the four different schools S_1, S_2, S_3, S_4 in a particular locality based upon the five different attributes D_1, D_2, D_3, D_4, D_5 which are listed with IVBPTrNNs. The decision maker has to select the best school among the four different schools based on the different attributes and also the different weightages based on the preference of attributes. The five different attributes are Curriculum, Fee Structure, Training for Higher Education, Vocational Training and Sports, which are analyzed with different weightages which are mentioned in the table below.[Table 1, Table 2].

Algorithm for PROMETHEE-II IVBPTrNNs: Preference Ranking Organization Method for Enrichment Evaluation[11].

Step 1 : According to the decision maker, an evaluation(decision) matrix is tabulated with IVBPTrNNs.

Step 2 : Score values to be calculated for each cell in the evaluation matrix.

Step 3 : Normalize the evaluation matrix for beneficial criteria and non-beneficial criteria by using the appropriate formulae:

Beneficial criteria: $R_{ij}=\frac{[x_{ij}-min(x_{ij})]}{[max(x_{ij})-min(x_{ij})]}$

Non-Beneficial criteria: $R_{ij}=\frac{[max(x_{ij})-x_{ij}]}{[max(x_{ij})-min(x_{ij})]}$, $i=1,2,...,m;\ j=1,2,...,n.$

Step 4 : Compute the evaluative differences of i^{th} choice in comparison to the others.

Step 5 : Set the preference function $P_j(a,b)$.

$P_j(a,b)=0$, if $R_{aj}\leq R_{bj}$

$P_j(a,b)=(R_{aj}-R_{bj})$, if $R_{aj}>R_{bj}$.

Step 6 : Calculate the aggregated preference function $\pi(a,b)=\frac{\left[\sum_{j=1}^{n}w_jP_j(a,b)\right]}{\sum_{j=1}^{n}w_j}$

Step 7 : Depending upon the number of alternatives, $m\times m$ matrix is formed and the aggregated preference value for the alternatives should be formed.

Step 8 : Determine the leaving and the entering outranking flows.

Leaving(positive) flow for a^{th} alternative, $\psi^{+}=\frac{1}{m-1}\sum_{b=1}^{m}\pi(a,b)$, $(a\neq b)$

Entering (negative) flow for a^{th} alternative, $\psi^{-}=\frac{1}{m-1}\sum_{b=1}^{m}\pi(b,a)$, $(a\neq b)$

Step 9 : Obtain each alternative's net outranking flow $\psi(a)=\psi^{+}(a)-\psi^{-}(a)$.

Step 10 : Determine the order in which all of the alternatives under consideration should be ranked based upon that value of $\psi(a)$. [Table 3, Table 4, Table 5, Table 6, Table 7, Table 8].

6 Conclusion

IVBPTrNNs will provide us with the detailed study of the attributes and alternatives which is mostly needed in the current scenario. The operational laws and the proposed score function for IVBPTrNNs were discussed and used to solve the illustrative example with more precision. We arrived with different rank preferences for different weightages under this comparative approach for IVBPTrNNs by PROMETHEE II method. Among all the different weightages, the highest weightage for curriculum and fee structure got the same ranking order of the schools $S_3 > S_1 > S_2 > S_4$, S_3 was chosen as the best school. IVBPTrNNs gives the decision maker to have an overall outlook of all the aspects and can make a better decision. Further study will be to extent the IVBPTrNNs concepts.

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